G2 AND G3 ADDITIONAL MATHEMATICS SYLLABUSES

Secondary Three to Four

Implementation starting with
2020 Secondary Three Cohort

Ministry of Education
SINGAPORE

CONTENT

SECTION 1: INTRODUCTION

Importance of Learning Mathematics
Secondary Mathematics Curriculum
Key Emphases

1. INTRODUCTION

Importance of Learning Mathematics

Mathematics contributes to the developments and understanding in many disciplines and provides the foundation for many of today's innovations and tomorrow's solutions. It is used extensively to model and understand real-world phenomena (e.g. consumer preferences, population growth, and disease outbreak), create lifestyle and engineering products (e.g. animated films, mobile games, and autonomous vehicles), improve productivity, decision-making and security (e.g. business analytics, academic research and market survey, encryption, and recognition technologies).

In Singapore, mathematics education plays an important role in equipping every citizen with the necessary knowledge and skills and the capacities to think logically, critically and analytically to participate and strive in the future economy and society. In particular, for future engineers and scientists who are pushing the frontier of technologies, a strong foundation in mathematics is necessary as many of the Smart Nation initiatives that will impact the quality of lives in the future will depend heavily on computational power and mathematical insights.

Secondary Mathematics Curriculum

Secondary education is a stage where students discover their strengths and interests. It is also the final stage of compulsory mathematics education. Students have different needs for and inclinations towards mathematics. For some students, mathematics is just a tool to be used to meet the needs of everyday life. For these students, formal mathematics education may end at the secondary levels. For others, they will continue to learn and need mathematics to support their future learning. For those who aspire to pursue STEM education and career, learning more advanced mathematics early will give them a head start.

For these reasons, the goals of the secondary mathematics education are:

- to ensure that all students will achieve a level of mastery of mathematics that will enable them to function effectively in everyday life; and
- for those who have the interest and ability, to learn more mathematics so that they can pursue mathematics or mathematics-related courses of study in the next stage of education.

There are 5 syllabuses in the secondary mathematics curriculum, catering to the different needs, interests and abilities of students:

- G3 Mathematics
- G2 Mathematics
- G1 Mathematics
- G3 Additional Mathematics
- G2 Additional Mathematics

The G3, G2 and G1 Mathematics syllabuses provide students with the core mathematics knowledge and skills in the context of a broad-based education. At the upper secondary levels, students who are interested in mathematics may offer Additional Mathematics as an elective. This prepares them better for courses of study that require mathematics.

Key Emphases

The key emphases of the 2020 syllabuses are summarised as follows:

1. Continue to develop in students the critical mathematical processes such as, reasoning, communication and modelling, as they enhance the learning of mathematics and support the development of 21st century competencies;

2. Develop a greater awareness of the nature of mathematics and the big ideas that are central to the discipline and bring coherence and connections between different topics so as to develop in students a deeper and more robust understanding of mathematics and better appreciation of the discipline; and

3. Give attention to developing students' metacognition by promoting self-directed learning and reflection.

SECTION 2:
MATHEMATICS CURRICULUM

Nature of Mathematics
Themes and Big Ideas
Mathematics Curriculum Framework
21st Century Competencies

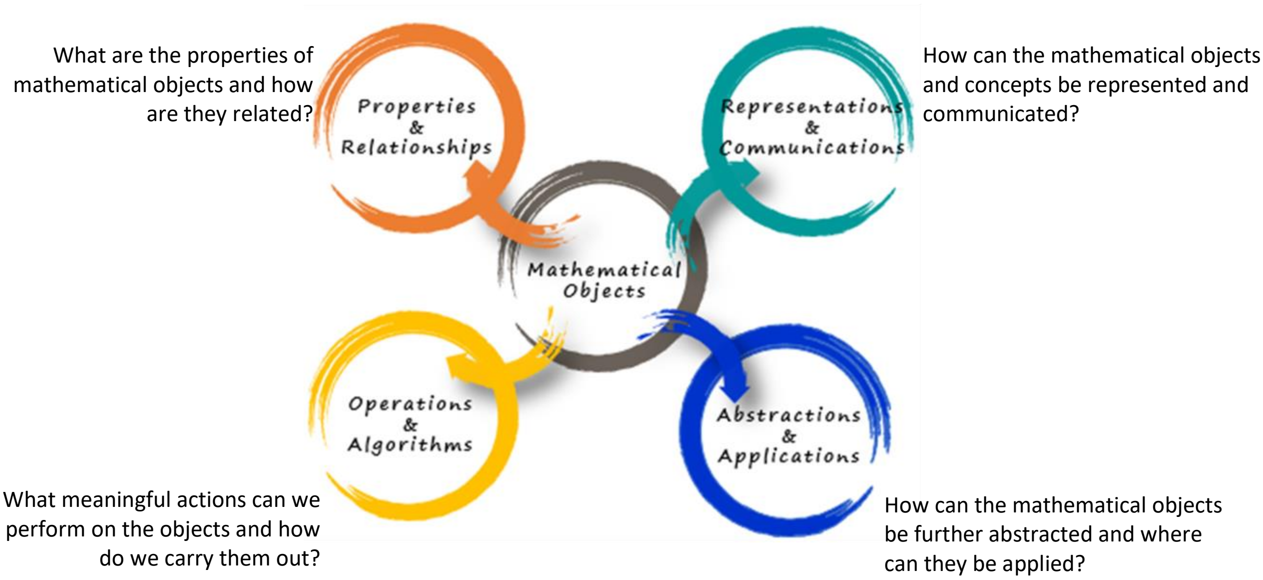
2. MATHEMATICS CURRICULUM

Nature of Mathematics

Mathematics can be described as a study of the *properties*, *relationships*, *operations*, *algorithms*, and *applications* of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. *Abstractions* are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for *representing* and *communicating* the ideas and results of the discipline.

Themes and Big Ideas

From the above description of the nature of mathematics, four recurring *themes* in the study of mathematics are derived.



1. *Properties and Relationships*: What are the properties of mathematical objects and how are they related?

   Properties of mathematical objects (e.g. numbers, lines, function, etc.) are either inherent in their definitions or derived through logical argument and rigorous proof. *Relationships* exist between mathematical objects. They include the proportional relationship between two quantities, the equivalence of two expressions or statements, the similarity between two figures and the connections between two functions. Understanding *properties and relationships* enable us to gain deeper insights into the mathematical objects and use them to model and solve real-world problems.

2. *Operations and Algorithms*: What meaningful actions can we perform on the mathematical objects and how do we carry them out?

Operations are meaningful actions performed on mathematical objects. They include arithmetic operations, algebraic manipulations, geometric transformations, operations on functions, and many more. *Algorithms* are generalised sequences of well-defined smaller steps to perform a mathematical operation or to solve a problem. Some examples are adding or multiplying two numbers and finding factors and prime numbers. Understanding the meaning of these *operations and algorithms* and how to carry them out enable us to solve problems mathematically.

3. *Representations and Communications*: How can the mathematical objects and concepts be represented and communicated within and beyond the discipline?

   Representations are integral to the language of mathematics. They include symbols, notations, and diagrams such as tables, graphs, charts and geometrical figures that are used to express mathematical concepts, properties and operations in a way that is precise and universally understood. *Communication* of mathematics is necessary for the understanding and dissemination of knowledge within the community of practitioners as well as general public. It includes clear presentation of proof in a technical writing as well as choosing appropriate representations (e.g. list, chart, drawing) to communicate mathematical ideas that can be understood by the masses.

4. *Abstractions and Applications*: How can the mathematical objects be further abstracted and where can they be applied?

   Abstraction is at the core of mathematical thinking. It involves the process of generalisation, extension and synthesis. Through algebra, we generalise arithmetic. Through complex numbers, we extend the number system. Through coordinate geometry, we synthesise the concepts across the algebra and geometry strands. The processes of abstraction make visible the structure and rich connections within mathematics and makes mathematics a powerful tool. *Application* of mathematics is made possible by abstractions. From simple counting to complex modelling, the abstract mathematical objects, properties, operations, relationships and representations can be used to model and study real-world phenomena.

Big ideas express ideas that are central to mathematics. They appear in different topics and strands. There is a continuation of the ideas across levels. They bring coherence and show connections across different topics, strands and levels. The big ideas in mathematics could be about one or more themes, that is, it could be about *properties and relationships* of mathematical objects and concepts and the *operations and algorithms* involving these objects and concepts, or it could be about *abstraction and applications* alone. Understanding the big ideas brings one closer to appreciating the nature of mathematics.

Eight clusters of big ideas are listed in this syllabus. These are not meant to be authoritative or comprehensive. They relate to the four themes that cut across and connect concepts from the different content strands, and some big ideas extend across and connect more concepts than others. Each cluster of big ideas is represented by a label e.g. big ideas about Equivalence, big ideas about Proportionality, etc.

Big Ideas about Diagrams

Main Themes: Representations and Communications

Diagrams are succinct, visual representations of real-world or mathematical objects that serve to communicate properties of the objects and facilitate problem solving. For example, graphs in coordinate geometry are used to represent the relationships between two sets of values, geometrical diagrams are used to represent physical objects, and statistical diagrams are used to summarise and highlight important characteristics of a set of data. Understanding what different diagrams represent, their features and conventions, and how they are constructed helps to facilitate the study and communication of important mathematical results.

Big Ideas about Equivalence

Main Themes: Properties and Relationships, Operations and Algorithms

Equivalence is a relationship that expresses the 'equality' of two mathematical objects that may be represented in two different forms. A number, algebraic expression or equation can be written in different but equivalent forms, and transformation or conversion from one form to another equivalent form is the basis of many manipulations for analysing and comparing them and algorithms for finding solutions.

Big Ideas about Functions

Main Themes: Properties and Relationships, Abstractions and Applications

A function is a relationship between two sets of objects that expresses how each element from the first set (input) uniquely determines (relates to) an element from the second set (output) according to a rule or operation. It can be represented in multiple ways, e.g. as a table, algebraically, or graphically. Functional relationships undergird many of the applications of mathematics and are used for modelling real-world phenomena. Functions are pervasive in mathematics and undergird many of the applications of mathematics and modelling of real-world phenomena.

Big Ideas about Invariance

Main Theme: Properties and Relationships, Operations and Algorithms

Invariance is a property of a mathematical object which remains unchanged when the object undergoes some form of transformation. In summing up or multiplying numbers, the sum or product is an invariant property that is not affected by the rearrangement of the numbers. In geometry, the area of a figure, the angles within it, and the ratio of the sides remain unchanged when the figure is translated, reflected or rotated. In statistics, the standard deviation remains unchanged when a constant is added to all the data points. Many mathematical results express invariance, e.g. a property of a class of mathematical objects.

Big Ideas about Measures

Main Theme: Abstractions and Applications

Numbers are used as measures to quantify a property of various real-world or mathematical objects, so that they can be analysed, compared, and ordered. There are many examples of measures such as length, area, volume, money, mass, time, temperature, speed, angles, probability, mean and standard deviation. Many measures have units, some measures have a finite range and special values which serve as useful references. In most cases, zero means the absence of the property while a negative measures the opposite property.

Big Ideas about Models

Main Themes: Abstractions and Applications, Representations and Communications

Models are abstractions of real-world situations or phenomena using mathematical objects and representations. For example, a real-world phenomenon may be modelled by a function, a real-world object may be modelled by a geometrical object, and a random phenomenon may be modelled by the probability distribution for different outcomes. As approximations, simplifications or idealisations of real-world problems, models come with assumptions, have limitations, and the mathematical solutions derived from these models need to be verified.

Big Ideas about Notations

Main Themes: Representations and Communications

Notations are symbols and conventions of writing used to represent mathematical objects, and their operations and relationships in a concise and precise manner. Examples include notations for mathematical constants like π and e, scientific notation to represent very big or very small numbers, set notations, etc. Understanding the meaning of mathematical notations and how they are used, including the rules and conventions, helps to facilitate the study and communication of important mathematical results, properties and relationships, reasoning and problem solving.

Big Ideas about Proportionality

Main Theme: Properties and Relationships

Proportionality is a relationship between two quantities that allows one quantity to be computed from the other based on multiplicative reasoning. Fraction, ratio, rate and percentage are different but related mathematical concepts for describing the proportional relationships between two quantities that allow one quantity to be computed from the other related quantity. In geometry, proportional relationships undergird important concepts such as similarity and scales. In statistics, proportional relationships are the basis for constructing and interpreting many statistical diagrams such as pie charts and histograms. Underlying the concept of proportionality are two quantities that vary in such a way that the ratio between them remains a constant.

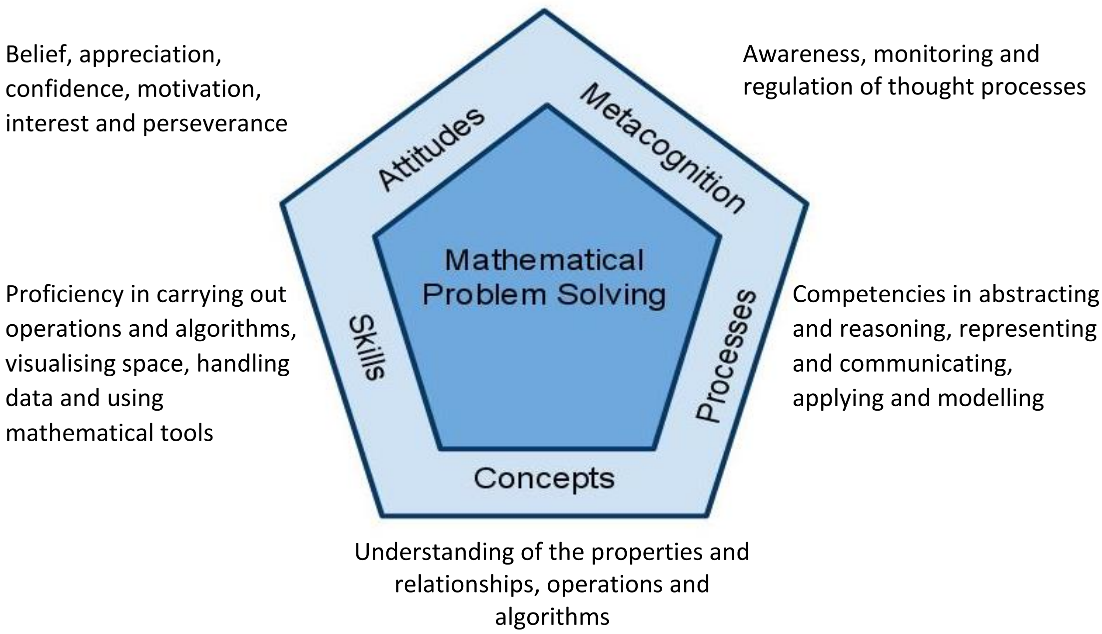
Mathematics Curriculum Framework

The central focus of the mathematics curriculum is the development of mathematical problem solving competency. Supporting this focus are five inter-related components – concepts, skills, processes, metacognition and attitudes.

Mathematical Problem Solving

Problems may come from everyday contexts or future work situations, in other areas of study, or within mathematics itself. They include straightforward and routine tasks that require selection and application of the appropriate concepts and skills, as well as complex and non-routine tasks that requires deeper insights, logical reasoning and creative thinking. General problem solving strategies e.g. Polya's 4 steps to problem solving and the use of heuristics, are important in helping one tackle non-routine tasks systematically and effectively.

Mathematics Curriculum Framework



Concepts

The understanding of mathematical concepts, their properties and relationships and the related operations and algorithms, are essential for solving problems. Concepts are organised by strands, and these concepts are connected and inter-related. In the secondary mathematics curriculum, concepts in numbers, algebra, geometry, probability and statistics and calculus (in Additional Mathematics) are explored.

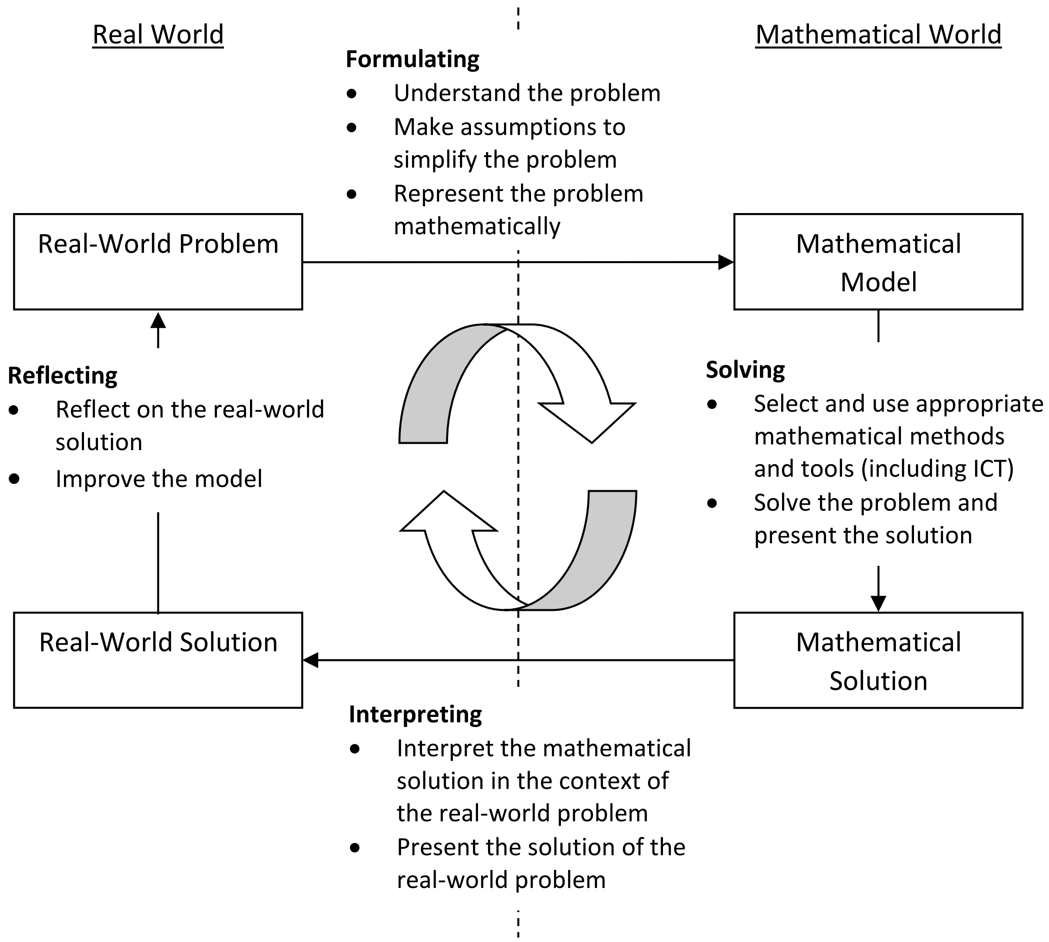
Skills

Being proficient in carrying out the mathematical operations and algorithms and in visualising space, handling data and using mathematical tools are essential for solving problems. In the secondary mathematics curriculum, operations and algorithms such as *calculation, estimation, manipulation*, and *simplification* are required in most problems. ICT tools such as spreadsheets, and dynamic geometry and graph sketching software may be used to support the learning.

Processes

Mathematical processes refer to the practices of mathematicians and users of mathematics that are important for one to solve problems and build new knowledge. These include abstracting, reasoning, representing and communicating, applying and modelling. Abstraction is what makes mathematics powerful and applicable. Justifying a result, deriving new results and generalising patterns involve reasoning. Expressing one's ideas, solutions and arguments to different audiences involves representing and communicating, and using the notations (symbols and conventions of writing) that are part of the mathematics language. Applying mathematics to real-world problems often involves modelling, where reasonable assumptions and simplifications are made so that problems can be formulated mathematically, and where mathematical solutions are interpreted and evaluated in the context of the real-world problem. The mathematical modelling process is shown in the diagram below.

Mathematical Modelling Process



Metacognition

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring and regulation of one's own thinking and learning. It also includes the awareness of one's affective responses towards a problem. When one is engaged in solving a non-routine or open-ended problem, metacognition is required.

Attitudes

Having positive attitudes towards mathematics contributes to one's disposition and inclination towards using mathematics to solve problems. Attitudes include one's belief and appreciation of the value of mathematics, one's confidence and motivation in using mathematics, and one's interests and perseverance to solve problems using mathematics.

Mathematics and 21st Century Competencies

The learning of mathematics creates opportunities for students to develop key competencies that are important in the 21st century. When students pose questions, justify claims, write and critique mathematical explanations and arguments, they are engaged in reasoning, critical thinking and communication. When students devise different strategies to solve an open-ended problem or formulate different mathematical models to represent a real-world problem, they are engaged in inventive thinking. When students vary their approaches to solve different but related problems, they are engaged in adaptive thinking.

As an overarching approach, the secondary mathematics curriculum supports the development of 21st century competencies (21CC) in the following ways:

1. The content are relevant to the needs of the 21st century. They provide the foundation for learning many of the advanced applications of mathematics that are relevant to today's world.

2. The pedagogies create opportunities for students to think critically, adaptively and inventively, reason logically and communicate effectively using mathematics, work individually as well as in groups, using ICT tools where appropriate in learning and doing mathematics.

3. The problem contexts raise students' awareness of local and global issues around them. For example, problems set around population, health and sustainability issues can help students understand the challenges faced by Singapore and those around the world.

The learning of mathematics also creates opportunities for students to apply knowledge, skills, and practices across STEM disciplines to solve real-world problems. Students can develop their curiosity, creativity, and agency to make a positive difference to the world. These goals of STEM learning i.e. be curious, be creative and be the change are closely linked to the 21CC.

SECTION 3:
G3 ADDITIONAL MATHEMATICS SYLLABUS

Aims of Syllabus
Syllabus Organisation
Applications and Contexts
Content

3 CONTENT: G3 ADDITIONAL MATHEMATICS SYLLABUS

Aims of Syllabus

The G3 Additional Mathematics syllabus aims to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, with emphasis in the sciences, but not limited to the sciences;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and
- appreciate the abstract nature and power of mathematics.

Syllabus Organisation

The concepts and skills covered in the syllabus are organised along 3 content strands. The development of processes, metacognition and attitudes are embedded in the learning experiences that are associated with the content.

<table>
<tr><td colspan="3">Concept and Skills</td></tr>
<tr><td>Algebra</td><td>Geometry and Trigonometry</td><td>Calculus</td></tr>
<tr><td colspan="3">Learning Experiences
(Processes, Metacognition and Attitudes)</td></tr>
</table>

Applications and Contexts

Solving problems in different contexts, including those in the sciences and engineering, should be part of the learning experiences of every student. These experiences give students the opportunities to apply the concepts and skills that they have learnt and to appreciate the value and power of mathematics.

Students will learn different functions, namely, linear, quadratic, exponential, logarithmic and trigonometric. These functions provide the building blocks for simple models. Students could be exposed to the following applications and contexts.

- Motion of projectile (quadratic functions and calculus)

- Optimisation problems e.g. maximising profits, minimising costs (functions and calculus)
- Population growth, radioactive decay, pH scale, Richter scale, decibel scale (exponential and logarithm functions)
- Financial mathematics e.g. profit and cost analysis, marginal profit (functions and calculus)
- Tidal waves, hours of daylight, simple harmonic motion (trigonometric functions)

The list above is by no means exhaustive or exclusive. Students are not required to have in-depth knowledge of these applications and contexts. Problems involving these contexts will provide sufficient information for students to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

Through the process of solving such problems, students will experience all or part of the mathematical modelling process. This includes:

- formulating the problem, including making suitable assumptions and simplifications;
- making sense of and discussing data, including real data presented as graphs and tables;
- selecting and applying the appropriate concepts and skills to solve the problem; and
- interpreting the mathematical solutions in the context of the problem.

Content

ALGEBRA

A1 Quadratic functions

1.1 Finding the maximum or minimum value of a quadratic function using the method of completing the square
1.2 Conditions for $y = ax^2 + bx + c$ to be always positive (or always negative)
1.3 Using quadratic functions as models

A2 Equations and inequalities

2.1 Conditions for a quadratic equation to have:
- two real roots
- two equal roots
- no real roots

and related conditions for a given line to:
- intersect a given curve
- be a tangent to a given curve
- not intersect a given curve

2.2 Solving simultaneous equations in two variables by substitution, with one of the equations being linear equation
2.3 Solving quadratic inequalities, and representing the solution on the number line

A3 Surds

3.1 Four operations on surds, including rationalising the denominator
3.2 Solving equations involving surds

A4 Polynomials and Partial Fractions

4.1 Multiplication and division of polynomials
4.2 Use of remainder and factor theorems, including factorising polynomials and solving cubic equations
4.3 Use of:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

4.4 Partial fractions with cases where the denominator is no more complicated than:
- $(ax+b)(cx+d)$
- $(ax+b)(cx+d)^2$
- $(ax+b)(x^2+c^2)$

A5 Binomial expansions

5.1 Use of the Binomial Theorem for positive integer n
5.2 Use of the notations $n!$ and $\binom{n}{r}$
5.3 Use of the general term

$\binom{n}{r} a^{n-r} b^r, 0 \le r \le n$

(knowledge of the greatest term and properties of the coefficients is not required)

A6 Exponential and Logarithmic Functions

6.1 Exponential and logarithmic functions $a^x, e^x, \log_a x, \ln x$ and their graphs, including
- laws of logarithms

- equivalence of $y = a^x$ and $x = \log_a y$
- change of base of logarithms

6.2 Simplifying expressions and solving simple equations involving exponential and logarithmic functions

6.3 Using exponential and logarithmic functions as models

GEOMETRY AND TRIGONOMETRY

G1 Trigonometric functions, identities and equations

1.1 Six trigonometric functions for angles of any magnitude (in degrees or radians)

1.2 Principal values of $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$

1.3 Exact values of the trigonometric functions for special angles $(30^\circ, 45^\circ, 60^\circ)$ or $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$

1.4 Amplitude, periodicity and symmetries related to sine and cosine functions

1.5 Graphs of $y = a\sin(bx) + c$, $y = a\sin\left(\frac{x}{b}\right) + c$, $y = a\cos(bx) + c$, $y = a\cos\left(\frac{x}{b}\right) + c$ and $y = a\tan(bx)$ where a is real, b is a positive integer and c is an integer

1.6 Use of:

$\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$,

$\sin^2 A + \cos^2 A = 1$,

$\sec^2 A = 1 + \tan^2 A$,

$\text{cosec}^2 A = 1 + \cot^2 A$

- the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
- the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$
- the expression of $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$

1.7 Simplification of trigonometric expressions

1.8 Solution of simple trigonometric equations in a given interval (excluding general solution)

1.9 Proofs of simple trigonometric identities

1.10 Use of trigonometric functions as models

G2 Coordinate geometry in two dimensions

2.1 Condition for two lines to be parallel or perpendicular

2.2 Midpoint of line segment

2.3 Area of rectilinear figure

2.4 Coordinate geometry of circles in the form:

- $(x - a)^2 + (y - b)^2 = r^2$
- $x^2 + y^2 + 2gx + 2fy + c = 0$

(excluding problems involving two circles)

2.5 Transformation of given relationships, including $y = ax^n$ and $y = kb^x$ to linear form to determine the unknown constants from a straight line graph

G3 Proofs in plane geometry

Use of:

3.1 properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles◆

3.2 congruent and similar triangles◆

3.3 midpoint theorem

3.4 tangent-chord theorem (alternate segment theorem)

◆ These are properties learnt in G3 Mathematics.

CALCULUS
C1 Differentiation and integration

1.1	Derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point
1.2	Derivative as rate of change
1.3	Use of standard notations $f'(x), f''(x), \frac{dy}{dx}, \frac{d^2y}{dx^2}\left[=\frac{d}{dx}\left(\frac{dy}{dx}\right)\right]$
1.4	Derivatives of x^n for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$, together with constant multiples, sums and differences
1.5	Derivatives of products and quotients of functions
1.6	Use of Chain Rule
1.7	Increasing and decreasing functions
1.8	Stationary points (maximum and minimum turning points and stationary points of inflexion)
1.9	Use of second derivative test to discriminate between maxima and minima
1.10	Apply differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems
1.11	Integration as the reverse of differentiation
1.12	Integration of x^n for any rational n, $\sin x$, $\cos x$, $\sec^2 x$ and e^x, together with constant multiples, sums and differences
1.13	Integration of $(ax+b)^n$ for any rational n, $\sin(ax+b)$, $\cos(ax+b)$ and e^{ax+b}
1.14	Definite integral as area under a curve
1.15	Evaluation of definite integrals
1.16	Finding the area of a region bounded by a curve and line(s) (excluding area of region between 2 curves)
1.17	Finding areas of regions below the x-axis
1.18	Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line

SECTION 4:
G2 ADDITIONAL MATHEMATICS SYLLABUS

Aims of Syllabus
Syllabus Organisation
Applications and Contexts
Content

4. CONTENT: G2ADDITIONAL MATHEMATICS SYLLABUS

Aims of Syllabus

The G2 Additional Mathematics syllabus aims to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, with emphasis in the sciences, but not limited to the sciences;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and
- appreciate the abstract nature and power of mathematics.

Syllabus Organisation

The concepts and skills covered in the syllabus are organised along 3 content strands. The development of processes, metacognition and attitudes are embedded in the learning experiences that are associated with the content.

Concept and Skills		
Algebra	Geometry and Trigonometry	Calculus
Learning Experiences (Processes, Metacognition and Attitudes)		

Applications and Contexts

Solving problems in different contexts, including those in the sciences and engineering, should be part of the learning experiences of every student. These experiences give students the opportunities to apply the concepts and skills that they have learnt and to appreciate the value and power of mathematics.

Students will learn different functions, namely, linear, quadratic and trigonometric. These functions provide the building blocks for simple models. Students could be exposed to the following applications and contexts.

- Motion of projectile (quadratic functions and calculus)
- Optimisation problems e.g. maximising profits, minimising costs (functions and calculus)

- Financial mathematics e.g. profit and cost analysis, marginal profit (functions and calculus)
- Tidal waves, hours of daylight, simple harmonic motion (trigonometric functions)

The list above is by no means exhaustive or exclusive. Students are not required to have in-depth knowledge of these applications and contexts. Problems involving these contexts will provide sufficient information for students to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

Through the process of solving such problems, students will experience all or part of the mathematical modelling process. This includes:

- formulating the problem, including making suitable assumptions and simplifications;
- making sense of and discussing data, including real data presented as graphs and tables;
- selecting and applying the appropriate concepts and skills to solve the problem; and
- interpreting the mathematical solutions in the context of the problem.

Content

ALGEBRA	
A1 Quadratic functions	
1.1	Finding the maximum or minimum value of a quadratic function using the method of completing the square
1.2	Conditions for $y = ax^2 + bx + c$ to be always positive (or always negative)
1.3	Using quadratic functions as models
A2 Equations and inequalities	
2.1	Conditions for a quadratic equation to have: • two real roots • two equal roots • no real roots and related conditions for a given line to: • intersect a given curve • be a tangent to a given curve • not intersect a given curve
2.2	Solving simultaneous equations in two variables by substitution, with one of the equations being linear equation
2.3	Solving quadratic inequalities, and representing the solution on the number line
A3 Surds	
3.1	Four operations on surds, including rationalising the denominator
3.2	Solving equations involving surds
A4 Polynomials and Partial Fractions	
4.1	Multiplication and division of polynomials
4.2	Use of remainder and factor theorems, including factorising polynomials and solving cubic equations
4.3	Use of: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
4.4	Partial fractions with cases where the denominator is no more complicated than: • $(ax+b)(cx+d)$ • $(ax+b)(cx+d)^2$ • $(ax+b)(x^2+c^2)$

GEOMETRY AND TRIGONOMETRY	
G1 Trigonometric functions, identities and equations	
1.1	Six trigonometric functions for angles of any magnitude (in degrees or radians)
1.2	Principal values of $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$
1.3	Exact values of the trigonometric functions for special angles $(30^\circ, 45^\circ, 60^\circ)$ or $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$
1.4	Amplitude, periodicity and symmetries related to sine and cosine functions
1.5	Graphs of $y = a\sin(bx) + c$, $y = a\sin\left(\frac{x}{b}\right) + c$, $y = a\cos(bx) + c$, $y = a\cos\left(\frac{x}{b}\right) + c$ and $y = a\tan(bx)$ where a is real, b is a positive integer and c is an integer

1.6	Use of: $\dfrac{\sin A}{\cos A} = \tan A$, $\dfrac{\cos A}{\sin A} = \cot A$, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\text{cosec}^2 A = 1 + \cot^2 A$ • the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ • the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ • the expression of $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$
1.7	Simplification of trigonometric expressions
1.8	Solution of simple trigonometric equations in a given interval (excluding general solution)
1.9	Proofs of simple trigonometric identities
1.10	Use of trigonometric functions as models
G2 Coordinate geometry in two dimensions	
2.1	Condition for two lines to be parallel or perpendicular
2.2	Midpoint of line segment
2.3	Area of rectilinear figure
2.4	Coordinate geometry of circles in the form: • $(x-a)^2 + (y-b)^2 = r^2$ • $x^2 + y^2 + 2gx + 2fy + c = 0$ (excluding problems involving two circles)
CALCULUS	
C1 Differentiation and integration	
1.1	Derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point
1.2	Derivative as rate of change
1.3	Use of standard notations $f'(x)$, $f''(x)$, $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}\left[=\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)\right]$
1.4	Derivatives of x^n, for any rational n, together with constant multiples, sums and differences
1.5	Derivatives of products and quotients of functions
1.6	Use of Chain Rule
1.7	Increasing and decreasing functions
1.8	Stationary points (maximum and minimum turning points and stationary points of inflexion)
1.9	Use of second derivative test to discriminate between maxima and minima
1.10	Apply differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems
1.11	Integration as the reverse of differentiation
1.12	Integration of x^n for any rational n, (excluding $n = -1$), together with constant multiples, sums and differences
1.13	Integration of $(ax+b)^n$ for any rational n, (excluding $n = -1$)
1.14	Definite integral as area under a curve
1.15	Evaluation of definite integrals
1.16	Finding the area of a region bounded by a curve and line(s) (excluding area of region between 2 curves)

Content for Sec 5 students taking G3 Additional Mathematics

[First Year of Exam: 2022]

This document listing the content for Sec 5 students taking G3 Additional Mathematics is to be read in conjunction with the G3 Additional Mathematics syllabus document.

Secondary Five

ALGEBRA

A5 Binomial expansions

5.1 Use of the Binomial Theorem for positive integer n

5.2 Use of the notations $n!$ and $\binom{n}{r}$

5.3 Use of the general term

$\binom{n}{r} a^{n-r} b^{r}, 0 \le r \le n$

(knowledge of the greatest term and properties of the coefficients is not required)

A6 Exponential and Logarithmic Functions

6.1 Exponential and logarithmic functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs, including

- laws of logarithms
- equivalence of $y = a^x$ and $x = \log_a y$
- change of base of logarithms

6.2 Simplifying expressions and solving simple equations involving exponential and logarithmic functions

6.3 Using exponential and logarithmic functions as models

GEOMETRY AND MEASUREMENT

G2 Coordinate geometry in two dimensions

The content 2.1 to 2.4 has been covered in G2 Additional Mathematics Syllabus.

2.5 Transformation of given relationships, including $y = ax^n$ and $y = kb^x$ to linear form to determine the unknown constants from a straight line graph

G3 Proofs in plane geometry

Use of:

3.1 properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles ♦

3.2 congruent and similar triangles ♦

3.3 midpoint theorem

3.4 tangent-chord theorem (alternate segment theorem)

♦ These are properties learnt in G3 Mathematics.

Secondary Five

CALCULUS

C1 Differentiation and integration

This topic in G2 Additional Mathematics syllabus

- is limited to derivatives and integrals of the standard function x^n (for any rational n), together with constant multiples, sums and differences
- does not include application of differentiation and integration to kinematics

1.4 Derivatives of $\sin x, \cos x, \tan x, e^x$ and $\ln x$, together with constant multiples, sums and differences

1.12 Integration of x^n (including $n = -1$), $\sin x, \cos x, \sec^2 x$ and e^x, together with constant multiples, sums and differences

1.13 Integration of $(ax+b)^n$ (including $n = -1$), $\sin(ax+b), \cos(ax+b)$ and $e^{(ax+b)}$

1.17 Finding areas of regions below the x-axis

1.18 Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line

SECTION 5:

TEACHING, LEARNING AND ASSESSING

Teaching Processes
Phases of Learning
Formative Assessment
Use of Technology and e-Pedagogy
Blended Learning
STEM Learning
Developing CT in Mathematics

5. TEACHING, LEARNING AND ASSESSING

Teaching Processes

The Pedagogical Practices of The Singapore Teaching Practice (STP) outlines four Teaching Processes that make explicit what teachers reflect on and put into practice before, during and after their interaction with students in all learning contexts.

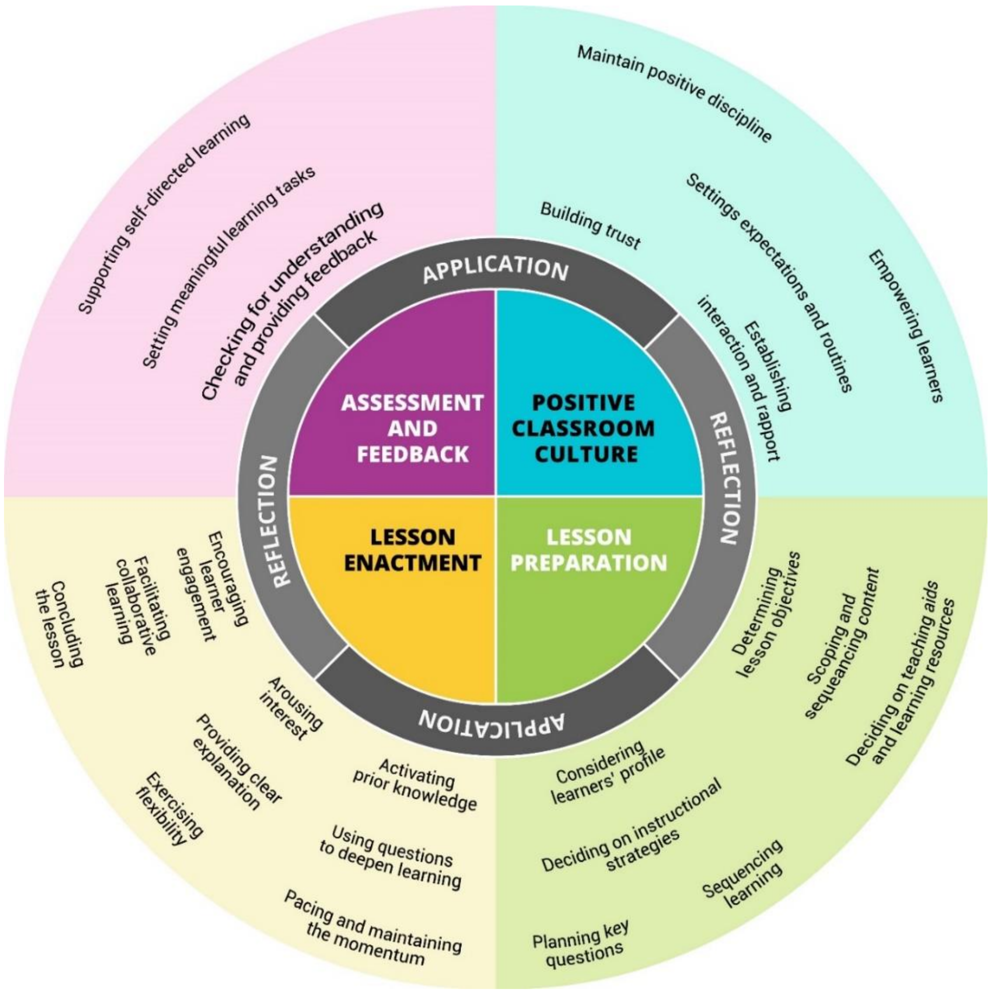
It is important to view the Pedagogical Practices of the STP in the context of the Singapore Curriculum Philosophy (SCP) and Knowledge Bases (KB), and also to understand how all three components work together to support effective teaching and learning.

Taking reference from the SCP, every student is valued as an individual, and they have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge, and skills. For learning to be effective, there is a need to adapt and match the teaching pace, approaches and assessment practices so that they are developmentally appropriate.

The 4 Teaching Processes are further expanded into Teaching Areas as follows:

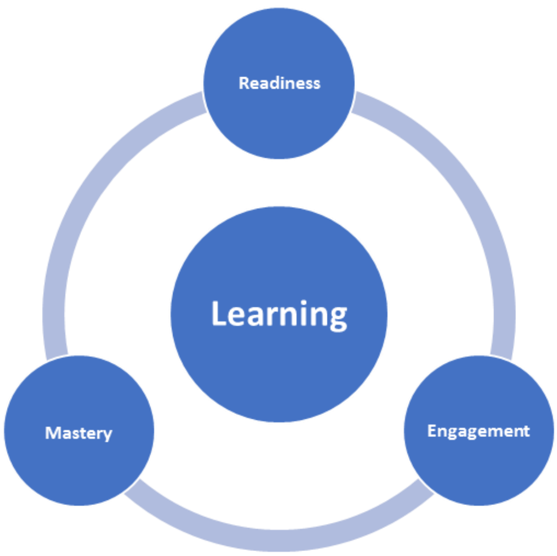
Assessment and Feedback • Checking for Understanding and Providing Feedback • Supporting Self-Directed Learning • Setting Meaningful Assignments	Positive Classroom Culture • Establishing Interaction and Rapport • Maintaining Positive Discipline • Setting Expectations and Routines • Building Trust • Empowering Learners
Lesson Enactment • Activating Prior Knowledge • Arousing Interest • Encouraging Learner Engagement • Exercising Flexibility • Providing Clear Explanation • Pacing and Maintaining Momentum • Facilitating Collaborative Learning • Using Questions to Deepen Learning • Concluding the Lesson	Lesson Preparation • Determining Lesson Objectives • Considering Learners' Profile • Selecting and Sequencing Content • Planning Key Questions • Sequencing Learning • Deciding on Instructional Strategies • Deciding on Teaching Aids and Learning Resources

The Teaching Areas are not necessarily specific to a single Teaching Process. Depending on the context, some of the Teaching Areas could be considered in another Teaching Process. The Teaching Processes are undergirded by a constant cycle of application and reflection.



Phases of Learning

The Teaching Areas in STP are evident in the effective planning and delivery of the three phases of learning - *readiness*, *engagement* and *mastery*.



Readiness Phase

Student readiness to learn is vital to learning success. Teachers have to consider the following:

- Learning environment
- Students' profile
- Students' prior and pre-requisite knowledge
- Motivating contexts

Engagement Phase

This is the main phase of learning where students engage with the new materials to be learnt (*Encouraging Learner Engagement*). As students have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge and skills, it is important to consider the pace of the learning and transitions (*Pacing and Maintaining Momentum*) using a repertoire of pedagogies.

Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. Teachers make deliberate choices on the instructional strategies (Deciding on Instructional Strategies) based on learners' profiles and needs, and the nature of the concepts to be taught. The engagement phase can include one or more of the following:

- Activity-based Learning
- Inquiry-based Learning
- Direct Instruction

Regardless of the approach, it is important for teachers to plan ahead, anticipate students' responses, and adapt the lesson accordingly (Exercising Flexibility).

Mastery Phase

The mastery phase is the final phase of learning where students consolidate and extend their learning. To consolidate, teachers summarise and review key learning points at the end of a lesson and make connections with the subsequent lesson (Concluding the Lesson). The mastery phase can include one or more of the following:

- Motivated Practice
- Reflective Review
- Extended Learning

Formative Assessment

Assessment is an integral part of the teaching and learning. It can be formative or summative or both. Formative assessment or Assessment for Learning (AfL) is carried out during teaching and learning to gather evidence and information about students' learning.

The *purpose* of formative assessment is to help students improve their learning and be self-directed in their learning. In learning of mathematics, just as in other subjects, information about students' understanding of the content must be gathered *before*, *during* and *after* the lesson.

The outcomes of the mathematics curriculum go beyond just the recall of mathematical concepts and skills. Since mathematical problem solving is the focus of the mathematics curriculum, assessment should also focus on students' understanding and ability to apply what they know to solve problems. In addition, there should be emphasis on processes such as reasoning, communicating, and modelling.

The overarching objectives of assessment should focus on students':

- understanding of mathematical concepts (going beyond simple recall of facts);
- ability to reason, communicate, and make meaningful connections and integrate ideas across topics;
- ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; and
- ability to develop strategies to solve non-routine problems.

The process of assessment is embedded in the planning of the lessons. The embedding of assessment process may take the following forms:

- Class Activities
- Classroom Discourse
- Individual or Group Tasks

Assessment provides feedback for both students and teachers.

- Feedback from teachers to students informs students where they are in their learning and what they need to do to improve their learning.
- Feedback from students to teachers comes from their responses to the assessment tasks designed by teachers. They provide information to teachers on what they need to do to address learning gaps, how to modify the learning activities students engage in, and how they should improve their instruction.
- Feedback between students is important as well because peer-assessment is useful in promoting active learning. It provides an opportunity for students to learn from each other and also allows them to develop an understanding of what counts as quality work by critiquing their peers' work in relation to a particular learning outcome.

Use of Technology

Computational tools are essential in many branches of mathematics. They support the discovery of mathematical results and applications of mathematics. Mathematicians use computers to solve computationally challenging problems, explore new ideas, form conjectures and prove theorems. Many of the applications of mathematics rely on the availability of computing power to perform operations at high speed and on a large scale. Therefore, integrating technology into the learning of mathematics gives students a glimpse of the tools and practices of mathematicians.

Computational tools are also essential for the learning of mathematics. In particular, they support the understanding of concepts (e.g. simulation and digital manipulatives), their properties (e.g. geometrical properties) and relationships (e.g. algebraic form versus graphical form). More generally, they can be used to carry out investigation (e.g., graphing tools), communicate ideas (e.g. presentation tools) and collaborate with one another as part of the knowledge building process (e.g. discussion forum). In particular, every student should be familiar with basic spreadsheet skills.

Getting students to design and implement some of the algorithms in mathematics (e.g. finding prime factors, multiplying two matrices, finding the median of a list of data) can potentially help these students develop a clearer understanding of the algorithms and the underlying mathematics concepts as well.

Blended Learning

Blended Learning transforms our students' educational experience by seamlessly blending different modes of learning. The key intents are to nurture: (i) self-directed and independent learners; and (ii) passionate and intrinsically motivated learners.

Blended Learning provides students with a broad range of learning experiences as shown in the diagram below. It includes the integration of *home-based learning (HBL) as a regular feature of the schooling experience*. Regular HBL can equip students with stronger abilities, dispositions and habits for independent and lifelong learning, in line with MOE's Learn for Life movement. Generally, all topics in the Secondary Mathematics syllabuses can be redesigned as a form of Blended Learning.

Examples of Blended Learning Experiences

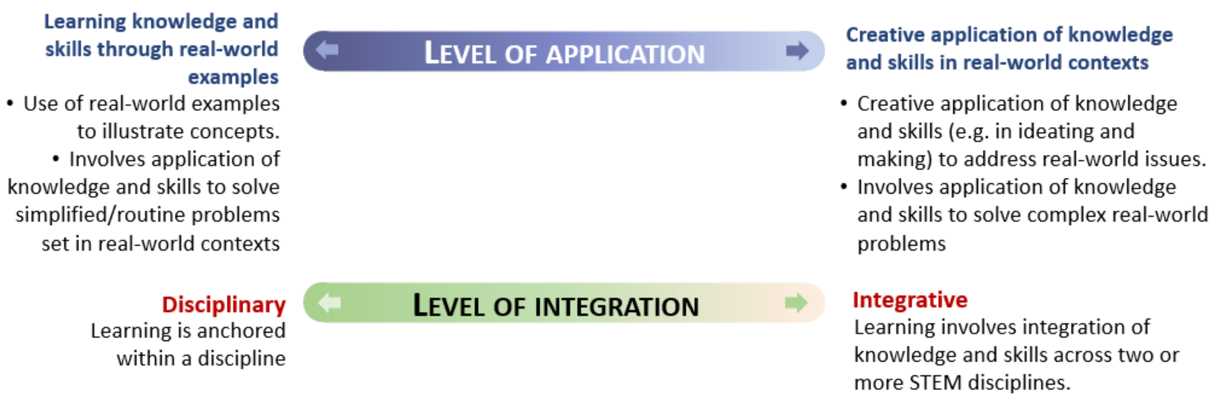
HBL also provides dedicated time and space for students to actively discover their interests and plan how they should go about pursuing them through student-initiated learning (SIL). There are three broad types of SIL activities, namely, school-curated, student-initiated with school facilitation and full student-initiated. These activities should be safe, wholesome and grounded on shared national values, and should engender a spirit of lifelong learning. Examples of SIL for Secondary Mathematics are exploring how Mathematics is used in an area of interest such as game designing and interior designing, and attempting real-world problems outside of the Mathematics curriculum.

For effective Blended Learning experiences, traditional in-class learning should be thoughtfully integrated with other learning approaches such as technology-based approaches. Teachers should be intentional and selective with the aspects of the curriculum to be delivered in school or at home, and leverage technology where it is meaningful and helpful for learning.

STEM Learning

STEM education seeks to strengthen the interest and capabilities of our students in STEM to prepare them for an increasingly complex and uncertain world. We want our students to be curious about the world around them, to think creatively and critically in solving problems, and be concerned citizens who make a difference in society. These are the goals of STEM education.

When designing STEM learning experiences, we can consider two aspects: 1) level of integration and 2) level of application. These two aspects lie on a continuum as illustrated in the figure below.



Design Considerations for STEM Learning

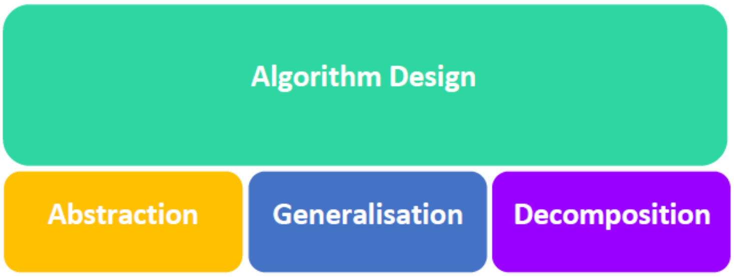
STEM learning experiences can happen at different levels of application,

- <u>within each STEM discipline:</u> Make learning relevant by selecting suitable real-world contexts to illustrate the knowledge, skills and practices in mathematics. Encourage creative application by allowing students to apply their mathematical knowledge, skills, and practices to work collaboratively in solving real-world problems and issues.
- <u>through integrating across STEM disciplines</u>: Enhance students' understanding to provide them with a more coherent and complete understanding of what they are learning in Mathematics by making connections with what is learnt in other STEM disciplines. Equip

students with the ability to manage complexity through learning experiences that require them to apply their knowledge, skills and practices across the STEM disciplines and work collaboratively in solving real-world problems with multiple solutions.

Developing Computational Thinking (CT) in Mathematics

Computational thinking can be described as the thought process involved in formulating problems and developing approaches to solving them in a manner that can be implemented with a computer (Wing, J. M., 2006). In general, computational thinking refers to four thinking skills, namely – abstraction, decomposition, generalisation and algorithm design, all of which are fundamental to problem-solving in mathematics.



Four Skills in Computational Thinking

The mathematics curriculum supports the development of CT by engaging students in tasks that involve algorithm design. Students will learn how to identify the essential information as input for their algorithm (abstraction), think of a general approach to solve the problem (generalisation) and to simplify the design of their algorithm by breaking them down into parts (decomposition) if necessary.

Designing algorithms, be it in words, flowcharts, or pseudo code, help students articulate their problem-solving approaches and make their thinking visible.

SECTION 6:
SUMMATIVE ASSESSMENT

Assessment Objectives
National Examinations

6. SUMMATIVE ASSESSMENT

Assessment Objectives

The purpose of summative assessments, such as tests and examinations, is to measure the extent to which students have achieved the learning objectives of the syllabuses.

The assessment objectives (AOs) reflect the emphasis of the syllabuses and describe what students should know and be able to do with the concepts and skills learned in each syllabus.

The AOs for the Additional Mathematics syllabuses are given below.

AOs	G3 and G2 Additional Mathematics
AO 1	**Use and apply standard techniques** • recall and use facts, terminology and notation • read and use information directly from tables, graphs, diagrams and texts • carry out routine mathematical procedures
AO 2	**Solve problems in a variety of contexts** • interpret information to identify the relevant mathematics concept, rule or formula to use • translate information from one form to another • make and use connections across topics/subtopics • formulate problems into mathematical terms • analyse and select relevant information and apply appropriate mathematical techniques to solve problems • interpret results in the context of a given problem
AO 3	**Reason and communicate mathematically** • justify mathematical statements • provide mathematical explanation in the context of a given problem • write mathematical arguments and proofs

National Examinations

Students will take national examination in their final year. The examination syllabuses can be found in the SEAB website. The following segment shows the national examination code and the scheme of assessment for the G3 and G2 Additional Mathematics syllabuses.

Scheme of Examination Papers

- G3 Additional Mathematics (Code – 4049, First Year of Examination – 2021)

Paper	Duration	Description	Marks	Weighting
1	2 h 15 min	There will be 12 – 14 questions of varying marks and lengths, up to 10 marks per question. Candidates are required to answer **all** questions.	90	50%
2	2 h 15 min	There will be 9 – 11 questions of varying marks and lengths, up to 12 marks per question. Candidates are required to answer **all** questions.	90	50%

- G2 Additional Mathematics (Code – 4051, First Year of Examination – 2021)

Paper	Duration	Description	Marks	Weighting
1	1 h 45 min	There will be 13 – 15 questions of varying marks and lengths. Candidates are required to answer **all** questions.	70	50%
2	1 h 45 min	There will be 8 – 10 questions of varying marks and lengths. Candidates are required to answer **all** questions.	70	50%