S4A TOPICAL INTENSIVE REVISION WEEK 1

Total Marks: 58

Topic: Equations and Inequalities

1(a) The equation of a curve is $y = 3x^2 + 5x + 1$. Find the set of values of x for which the curve lies completely above the line $y - 3x = 2$. [3]

(b) Find the range of values of m for which the equation $mx^2 + 2m = 3x(4 - x)$ has real roots. [3]

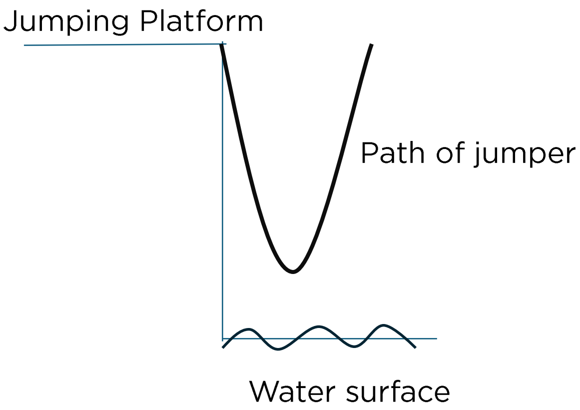
2(a) Find the set of values of x for which $5x^2 + 12x > 3x + 2$. [3]

(b) The line $y = mx + c$ does not intersect the curve $x^2 + y^2 = 8$.

(i) Prove that $8m^2 + 8 < c^2$ [4]

(ii) Hence, determine whether the line $y = 2x + 5$ intersects the curve $x^2 + y^2 = 8$ [2]

3. The path of a typical bungee jumper is described by a quadratic curve as shown in the diagram



This quadratic curve can be modelled by the equation $y = 50x^2 - 100x + 58$, where y is the vertical height, in metres, from the water surface and x is the horizontal distance, in metres, from the jumping platform.

(a) State the height of the jumping platform above the water surface. [1]

(b) Express $50x^2 - 100x + 58$ in the form $a(x + b)^2 + c$. [3]

Assuming you are a safety engineer looking into the design of this bungee jumping facility,

(c) By using your answer to (b) and without solving for x, explain how you might prove that a bungee jumper will never hit the water surface. [3]

4(a) The expression $f(x) = ax^3 + (a - 3b)x^2 + 3bx + c$ is exactly divisible by $x^2 + 3x$. When $f(x)$ is divided by $x + 2$, the remainder is 10. Find the values of a, b and c. Hence factorise $f(x)$ completely. [6]

(b) Given that $3x^3 - 2x^2 + x - 4 = A(x-1) + B(x-1)(x+1) + Cx(x^2 - 1) + D$ for all values of x, find the values of A, B, C and D. [4]

5. It is given that $f(x) = 2x^3 + ax^2 + x + b$.

(i) Find the value of a and of b for which $2x^2 + x - 1$ is a factor of $f(x)$. [4]

(ii) Solve the equation $f(x) = 0$. [2]

(iii) Hence, solve $\frac{1}{4}y^3 + \frac{a}{4}y^2 + \frac{1}{2}y + b = 0$. [2]

6. The polynomial $f(x)$ is such that the coefficient of x^4 is 5. The roots of the equation are $f(x) = 0$ are $\frac{1}{5}$ and 2. $f(x)$ has a remainder of -12 when divided by $x - 1$ and a remainder of 162 when divided by $x + 1$.

(i) Find an expression for $f(x)$. [4]

(ii) Prove that the polynomial $f(x) = 0$ has only two solutions. [2]

7(a) Factorise $3x^3 - 24y^3$ completely. [2]

(b) Express $\frac{7x^2+19x+15}{(x+1)^2(x+2)}$ as partial fractions. [4]

8. Express $\frac{3x^3+9}{x^3-9x}$ in partial fractions. [6]

Answer Key

1a.	$x < -1$ or $x > \frac{1}{3}$
1b.	$-6 \leq m \leq 3$
2a.	$x < -2$ or $x > \frac{1}{5}$
2bii.	Intersect
3a.	58
3b.	$y = 50(x-1)^2 + 8$
3c.	Since the minimum height from the water surface is 8m, which is greater than 0, the bungee jumper will never hit the surface
4a.	$a = 2, b = -1, c = 0, f(x) = x(x+3)(2x-1)$
4b.	$A = 4, B = -2, C = 3, D = -2$
5i.	$a = 5, b = -2$
5ii.	$x = 0.5, -1, -2$
5iii.	$y = 1, -2, -4$
6i.	$f(x) = (5x-1)(x-2)(x^2-3x+5)$
7a.	$3(x-2y)(x^2+2xy+4y^2)$
7b.	$\frac{2}{x+1} + \frac{3}{(x+1)^2} + \frac{5}{x+2}$
8.	$\frac{3x^3+9}{x^3-9x} = 3 - \frac{1}{x} - \frac{4}{x+3} + \frac{5}{x-3}$