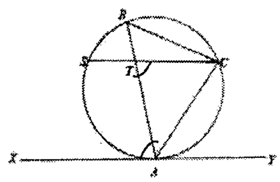
2022 Prelim A Maths Paper 1 Marking Scheme

Qn	Solution	Marks	Remarks
1	$y = x^2 + 3x - 8$ —— (1) $y = \frac{3}{2}x - \frac{7}{2}$ —— (2) (1) = (2)		
	$x^2 + 3x - 8 = \frac{3}{2}x - \frac{7}{2}$	[M1]	
	$2x^2 + 3x - 9 = 0$		
	$(2x-3)(x+3) = 0$	[M1]	
	$x = \frac{3}{2}$ or $x = -3$	[M1]	
	$y = -\frac{5}{4}$ or $y = -8$ Coord of A and B are $\left(\frac{3}{2}, -\frac{5}{4}\right)$ & $(-3, -8)$. Distance btw A and B $= \sqrt{(1.5+3)^2 + (-1.25+8)^2}$ $= 8.11$ units	[A1]	Accept exact surd $\frac{\sqrt{1053}}{4}$
2	$\frac{7x+8}{(2x+1)(x-1)^2} = \frac{A}{2x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$	[M1]	
	$7x+8 = A(x-1)^2 + B(2x+1)(x-1) + C(2x+1)$	[M1]	
	Let $x = 1$, $15 = 3C$, $C = 5$ Let $x = -\frac{1}{2}$, $\frac{9}{2} = A\left(-\frac{3}{2}\right)^2$, $A = 2$ Let $x = 0$, $8 = 2(-1)^2 + B(1)(-1) + 5(1)$, $B = -1$	[A3]	
	$\frac{7x+8}{(2x+1)(x-1)^2} = \frac{2}{2x+1} - \frac{1}{x-1} + \frac{5}{(x-1)^2}$		

3	$a+b\sqrt{3}=\dfrac{\left(4-\sqrt{3}\right)^2}{2+\sqrt{3}}$ $=\dfrac{19-8\sqrt{3}}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $=\dfrac{38-19\sqrt{3}-16\sqrt{3}+24}{2^2-3}$ $=62-35\sqrt{3}$ $a=62,\quad b=-35$	[M1] [M1] [A1]	Accept alternative method $\left(a+b\sqrt{3}\right)\left(2+\sqrt{3}\right)=\left(4-\sqrt{3}\right)^2$
4	$y=a(x+2)^2+k$ At $(-1,4)$, $4=a(-1+2)^2+k$ $a+k=4$ Since $a>0$ & $k>0$ as curve lies above x-axis, let $a=1$, then $k=3$ A possible equation for the curve is $y=(x+2)^2+3$	[M1] [M1] [M1] [A1]	(Any other suitable equation satisfying the relevant conditions accepted)
5(a) (b) (c)	$r=\dfrac{\max+\min}{2}=\dfrac{1+(-3)}{2}=-1$ (shown) Period of curve $=\dfrac{2\pi}{\frac{1}{q}}=8\pi,\quad q=4$ Amplitude $=\dfrac{\max-\min}{2}=\dfrac{1-(-3)}{2}=2,\ p=-2$ Equation of the curve is $y=-2\sin\dfrac{x}{4}-1$	[B1] [B1] [B1] [B1]	
6(a)	$f'(x)=\int(6x+2)\,dx$ $=3x^2+2x+c$ At $x=-1$, $f'(x)=11$ $11=3(-1)^2+2(-1)+c$ $c=10$ $f'(x)=3x^2+2x+10$	[M1] [M1] [A1]	

(b)	$f(x)=\int(3x^2+2x+10)\,dx$ $=x^3+x^2+10x+d$ At $(-1,10)$, $10=(-1)^3+(-1)^2+10(-1)+d$ $d=20$ $f(x)=x^3+x^2+10x+20$	[M1] [A1]	
(c)	For $y=f(x)$ to have stationary points, set $f'(x)=0$. $3x^2+2x+10=0$ $b^2-4ac=2^2-4(3)(10)=-116<0$ $f'(x)=0$ has no real solution, ie no stationary points.	[M1] [M1] [A1]	Solve equation to show no real roots accepted No marks if no real roots is not mentioned
7(a)	$x=4.5,\quad V=\frac{1}{3}\pi(4.5)^2(18-4.5)$ $=\frac{729}{8}\pi$ $\frac{dV}{dt}=\frac{\frac{729}{8}\pi}{9}=\frac{81}{8}\pi$ cm/s (or $10\frac{1}{8}\pi$ cm/s)	[M1] [M1.A1]	Accept 10.125π *or* 31.8 cm/s
(b)	$V=\frac{\pi}{3}(18x^2-x^3)$ $\frac{dV}{dx}=\frac{\pi}{3}(36x-3x^2)$ *At* $x=4.5$, $\frac{dV}{dx}=\frac{\pi}{3}(36(4.5)-3(4.5)^2)$ $=\frac{135}{4}\pi$ Using $\frac{dV}{dt}=\frac{dV}{dx}\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{\frac{81}{8}\pi}{\frac{135}{4}\pi}=0.3$ The water level is rising at a rate of 0.3 cm/s	[M1] [M1] [M1. A1]	

8(a)	$\sin^3 x + \cos^3 x$ $= (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)$	[B1]	
(b)	$LHS = \dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x}$ $= \dfrac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\sin x + \cos x}$ $= \sin^2 x - \frac{1}{2}(2 \sin x \cos x) + \cos^2 x$ $= 1 - \frac{1}{2}\sin 2x = RHS$	[M1] [A1, A1]	No mark awarded if student do not show $\sin x \cos x = \frac{1}{2}(2 \sin x \cos x)$
(c)	$1 - \frac{1}{2}\sin 2x = \frac{5}{4}$ $\frac{1}{2}\sin 2x = -\frac{1}{4}$ $\sin 2x = -\frac{1}{2}$ Basic angle, $\alpha = 30^o$ $2x = 210^o, 330^o, 570^o, 690^o$ $x = 105^o, 165^o, 285^o, 345^o$	[M1] [M1] [M1] [A1]	
9(a)	$\angle ACB = \angle ATC$ (given) $\angle BAC = \angle CAT$ (common) $\therefore \Delta ABC \,\&\, \Delta ACT$ are similar (AA Test)	[M1] [A1]	Either statement
(b)	Since $\Delta ABC \,\&\, \Delta ACT$ are similar, $\dfrac{AB}{AC} = \dfrac{AC}{AT}$ $AC^2 = (AB)(AT)$ $= (AT + TB)(AT)$ $= AT^2 + AT \times TB$ $AC^2 - AT^2 = AT \times TB$ (shown)	[M1] [M1] [A1]	
(c)	$\angle BAX = \angle ACB$ (alt. segment thm) $\& \angle ACB = \angle ATC$ (given) $\therefore \angle BAX = \angle ATC$ By the alternate angle property, SC and XY are parallel.	[M1] [M1] [A1]	

10(a)	$m_{AB}=\frac{p-10}{3}$ $m_{CB}=\frac{6-p}{1}$ Since $\angle ABO=\angle CBO, m_{AB}=-m_{CB}$ $\frac{p-10}{3}=-(6-p)$ $p-10=-18+3p$ $-2p=-8$ $p=4$ (shown)	[M1] [M1] [A1]	Accept method involving tan θ
(b)	$m_{AB}=-2$ $m_{AD}=\frac{1}{2}$ Equation of line *AD* is $y-10=\frac{1}{2}(x+3)$ $y=\frac{1}{2}x+\frac{23}{2}$ (shown) $CD \parallel AB,\ m_{CD}=-2$ Equation of line *CD* is $y-6=-2(x-1)$ $y=-2x+8$ At D, $\frac{1}{2}x+\frac{23}{2}=-2x+8$ $\frac{5}{2}x=-\frac{7}{2}$ $x=-\frac{7}{5}, y=-2\left(-\frac{7}{5}\right)+8=\frac{54}{5}$ Coord of D is $\left(-\frac{7}{5},\frac{54}{5}\right)$	[M1] [A1] [M1] [M1] [A1]	Accept (-1.4, 10.8)

(c)	Area of $ABCD$ $=\frac{1}{2}\begin{vmatrix} 0 & 1 & -1.4 & -3 & 0 \\ 4 & 6 & 10.8 & 10 & 4 \end{vmatrix}$ $==\frac{1}{2}\left[(0+10.8-14-12)-(4-8.4-32.4)\right]$ $=10.8\text{ units}^2$	[M1] [A1]	
11(a)	$x^2+4x+3=(x+3)(x+1)$ By Factor Theorem, $P(-3)=0 \quad \& \quad P(-1)=0$ $5(-3)^3+a(-3)^2+3+b=0$ $9a+b=132 \quad ----(1)$ $5(-1)^3+a(-1)^2+1+b=0$ $a+b=4 \quad ----(2)$ $(1)-(2): \quad 8a=128$ $a=16, \quad b=-12$	[M1] [M1] [M1] [A1]	*overall minus 1 mark if any expression is not set to 0
(b)	$5x^3+16x^2-x-12=(x+3)(x+1)(px+q)$ By comparison, $p=5 \quad \& \quad q=-4$ $P(x)=0$ $(x+3)(x+1)(5x-4)=0$ $x=-3 \text{ or } -1 \text{ or } \frac{4}{5}$	[M1] [M1] [A1]	
(c)	$5(x^2)^3+a(x^2)^2-x^2+b=0$ ---- (1) Let $u=x^2$, (1) becomes $5u^3+au^2-u+b=0$ From (b), $u=-3(rej) \quad or \quad -1(rej) \quad or \quad \frac{4}{5}$ $\therefore x^2=\frac{4}{5}$ $x=\frac{2}{\sqrt{5}} \text{ or } -\frac{2}{\sqrt{5}}$	[M1] [A1]	Accept $\pm\frac{2\sqrt{5}}{5}$

12(a)	$y=\frac{7}{6x+1}=7(6x+1)^{-1}$ $\frac{dy}{dx}=-7(6x+1)^{-2}(6)$ $=-\frac{42}{(6x+1)^2}$ At K, $x=1$, $\frac{dy}{dx}=-\frac{42}{7^2}=-\frac{6}{7}$ Equation of tangent at K is $y-1=-\frac{6}{7}(x-1)$ $y=-\frac{6}{7}x+\frac{13}{7}$ Coord of B is $\left(0,\frac{13}{7}\right)$	[M1] [M1] [M1] [M1] [A1]	
(b)	Area of the shaded region = Area under curve – Area of trapezium $=\int_0^1 \frac{7}{6x+1}dx-\frac{1}{2}(1)\left(1+\frac{13}{7}\right)$ $=7\left[\frac{1}{6}\ln(6x+1)\right]_0^1-\frac{10}{7}$ $=\frac{7}{6}(\ln 7-\ln 1)-\frac{10}{7}$ $=0.842$ units^2	[M1,,M1] [M1] [M1] [A1]	
13(a)	$(\ln x)^2+\frac{2}{\frac{\ln e}{\ln x}}=3$ $(\ln x)^2+2\ln x-3=0$ Let $u=\ln x$ $u^2+2u-3=0$ $(u+3)(u-1)=0$ $u=-3$ or $u=1$ $\ln x=-3$ or $\ln x=1$ $x=e^{-3}=\frac{1}{e^3}$ or $x=e$	[M1] [M1] [M1] [A1]	Accept $x=0.0498$ or $x=2.72$

(b)	$\lg\left(\frac{p}{2q}\right)=\lg(p+2q)$ $\frac{p}{2q} = p+2q$ (i) $p = 2pq+4q^2$ $p(1-2q)=4q^2$ $p=\frac{4q^2}{1-2q}$	[M1] [A1]	
	(ii) Range of p is $p>0$ $\therefore \frac{4q^2}{1-2q}>0$ Since $q>0$ for lg $2q$ to be defined, $4q^2>0$ & $1-2q>0$ $q<\frac{1}{2}$ $\therefore 0<q<\frac{1}{2}$ (shown)	[B1] [B1]	
(c)	By observing the shape of the curve, a logarithmic function, ie equation (B) $y=a\ln x+b$ is a suitable model since the rate of growth of the head circumference gets much slower when the baby gets older over the months.	[B1] [B1]	-for correct model -for correct reasoning