Name : ___Solutions___

Class	Index Number

METHODIST GIRLS' SCHOOL
Founded in 1887

PRELIMINARY EXAMINATION 2021
Secondary 4

Wednesdaday	**MATHEMATICS**	4048/02
4 August 2021	**Paper 2**	2 h 30 min

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For , use either your calculator value or 3.142, unless the question requires the answer in terms of

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

100
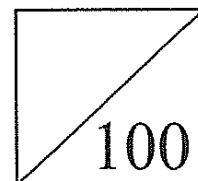
This question paper consists of 21 printed pages and 3 blank pages.

2

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

3

1 **(a)** Simplify $\dfrac{25m^2-4}{4-4m-15m^2}$.

$$\dfrac{25m^2-4}{4-4m-15m^2}$$
$$=\dfrac{(5m-2)(5m+2)}{(2-5m)(2+3m)}$$
$$=\dfrac{(5m-2)(5m+2)}{-(5m-2)(2+3m)}$$
$$=-\dfrac{5m+2}{2+3m}$$

Answer .... [3]

(b) Given that $r=3-\sqrt{\dfrac{p^2}{4}-q}$,

(i) evaluate r when $p=-8$ and $q=7$,

$$r=3-\sqrt{\dfrac{(-8)^2}{4}-7}$$
$$=0$$

Answer .... [1]

(ii) express p in terms of q and r.

$$r=3-\sqrt{\dfrac{p^2}{4}-q}$$
$$\dfrac{p^2}{4}-q=(3-r)^2$$
$$p=\pm2\sqrt{(3-r)^2+q}$$

Answer .... [3]

(c) The value of a 2-digit number can be written as $10x + y$ where x is the tens digit of the number and y is its units digit.

(i) When the digits x and y are reversed, the value is increased by 27. Show that $y - x = 3$. [1]

$$10y + x - (10x + y) = 27$$
$$9y - 9x = 27$$
$$y - x = 3$$

(ii) The sum of the original number and the number with reverse digits is 121. Show that $x + y = 11$. [1]

$$10y + x + 10x + y = 121$$
$$11y + 11x = 121$$
$$y + x = 11$$

(iii) Hence, find the original number.

$$y - x + y + x = 14$$
$$y = 7$$
$$x = 4$$

The number is 47.

Answer .... [3]

2 Bag A has 4 green counters, 6 red counters and 2 yellow counters.

(a) Alice takes a counter at random from Bag A, notes its colour and replaces it in the bag. She repeated this process n times. Write down an expression, in terms of n, for the probability that she takes only green counter.

$$\left(\frac{1}{3}\right)^n$$

Answer .... [1]

(b) Bill takes a counter at random from Bag A, notes its colour and replaces it back. He then takes a second counter at random from Bag A.

Calculate the probability that Bill takes only one yellow counter.

$$\frac{1}{6}\times\frac{5}{6}\times 2$$
$$=\frac{5}{18}$$

Answer .... [2]

Bag B has 3 green counters and 3 red counters.

(c) Charles takes a counter at random from Bag A and places it into Bag B.
He then takes a counter at random from Bag B and places it into Bag A.
Find the probability that after these two moves, Bag A has 4 green counters, 6 red counters and 2 yellow counters.

$$\frac{1}{3}\times\frac{4}{7}+\frac{1}{2}\times\frac{4}{7}+\frac{1}{6}\times\frac{1}{7}$$
$$=\frac{1}{2}$$

Answer .... [3]

6

3 (a) The table shows the height, h cm, of 60 plants.

Height (h cm)	$10 < h \le 20$	$20 < h \le 30$	$30 < h \le 40$	$40 < h \le 50$
Frequency	7	14	27	12

(i) Calculate an estimate of the standard deviation of the height of the plants.

9.10 (3sf)

Answer ...:...................... cm [1]

(ii) Two plants are chosen at random. Find the probability that both plants have height greater than 30 cm.

$\dfrac{39}{60} \times \dfrac{38}{59}$

$= \dfrac{247}{590}$

Answer .... [2]

(b) A toothpaste supplier delivers tubes of toothpaste to 3 different shops A, B and C. The number of tubes of toothpaste supplied per delivery to each shop, the sizes and sale prices of the tubes, together with the number of deliveries made to each shop over a 6-month period are shown in the table below.

Size of tube		Number of tubes of toothpaste per delivery			Number of deliveries over 6 months
		small	medium	large	
Name of shop	A	68	85	105	4
	B	75	–	62	7
	C	–	130	98	5
Sale price per tube		$2.50	$4.20	$6.00	

It is given that $\mathbf{T} = \begin{pmatrix} 68 & 85 & 105 \\ 75 & 0 & 62 \\ 0 & 130 & 98 \end{pmatrix}$.

(i) Write down the matrix $\mathbf{P}$ such that $\mathbf{TP}$ gives the total number of tubes of toothpaste delivered to each shop per delivery.

$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Answer $\mathbf{P} =$ .... [1]

(ii) **(a)** Write down a 1 × 3 matrix **D** to represent the number of deliveries over 6 months to each shop.

$$\begin{pmatrix} 4 & 7 & 5 \end{pmatrix}$$

Answer   **D** = ... [1]

(ii) **(b)** Evaluate **DT**.

$$\begin{pmatrix} 797 & 990 & 1344 \end{pmatrix}$$

Answer   **DT** = ... [1]

(c) State what the elements of **DT** represent.

Answer

It represents the <u>total</u> number of tubes of toothpaste of <u>each size</u> delivered to <u>all the shops over 6 months</u>.

[1]

(iii) **Q** is a matrix that represents the total amount of money that would be obtained from each shop from the sale of all the tubes of toothpaste per delivery.
By matrix multiplication, find **Q**.

$$\begin{pmatrix} 68 & 85 & 105 \\ 75 & 0 & 62 \\ 0 & 130 & 98 \end{pmatrix}\begin{pmatrix} 2.5 \\ 4.2 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} 1157 \\ 559.5 \\ 1134 \end{pmatrix}$$

Answer   **Q** = ... [2]

8

4 (a) Alan drove a distance of x km and his car used 7.5 litres of fuel.
Write down an expression, in terms of x, for the rate of fuel used by his car in litres per 100 km.

$$\frac{750}{x}$$

Answer .... litres per 100 km [1]

(b) Ben drove a distance of $(x + 30)$ km and his car used 8 litres of fuel.
Write down an expression, in terms of x, for the rate of fuel used by his car in litres per 100 km.

$$\frac{800}{x+30}$$

Answer .... litres per 100 km [1]

(c) The rate of fuel used by Alan's car is 1.5 litres per 100 kilometre more than the rate of fuel used by Ben's car.
Form an equation to represent this information and show that it can be simplified to $3x^2 + 190x - 45000 = 0$.

Answer [3]

$$\frac{750}{x} - \frac{800}{x+30} = \frac{3}{2}$$
$$2[750(x+30) - 800x] = 3x(x+30)$$
$$45000 - 100x = 3x^2 + 90x$$
$$3x^2 + 190x - 45000 = 0$$

(d) Solve the equation $3x^2 + 190x - 45000 = 0$, giving both answers correct to two decimal places.

$$\frac{-190 \pm \sqrt{190^2 - 4(3)(-45000)}}{2(3)}$$

$= 94.8354 \quad$ or $\quad -158.1687$

$= 94.84$ (2dp) $\quad = -158.17$ (2dp)
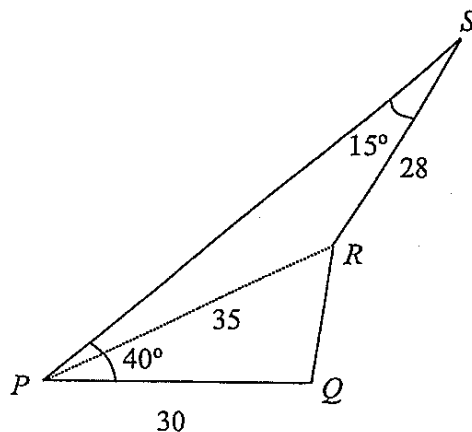
Answer $x =$ .... or .... [3]

(e) Each litre of fuel costs $2.42.
Calculate the amount Alan paid for the petrol if he drove 120 km.

$$\frac{7.5}{94.8354} \times 120 \times 2.42$$

$= \$22.97$

Answer $ .... [2]

5   The diagram shows four points P, Q, R and S on level ground.
     Q is due east of P.
     Angle $SPQ = 40°$ and angle $PSR = 15°$.
     $PQ = 30$ m, $PR = 35$ m and $RS = 28$ m.

(a)   Calculate

(i)   the bearing of R from P,

$$\frac{\sin \angle SPR}{28} = \frac{\sin 15°}{35}$$

$\angle SPR = 11.9498°$

$\angle RPQ = 40° - 11.9498° = 28.0502°$

Bearing $= 90° - 28.0502° = 061.9°$ (1dp)

Answer ....° [3]

(ii) QR,

$QR^2 = 30^2 + 35^2 - 2(30)(35)\cos 28.0502°$

$QR = \sqrt{30^2 + 35^2 - 2(30)(35)\cos 28.0502°}$

$QR = 16.5$ m (3sf)

Answer .... m [2]

(iii) the area of triangle *PRS*.

$$\frac{1}{2} \times 35 \times 28 \times \sin 153.0502°$$
$$= 222 \text{ m}^2 \text{ (3sf)}$$

Answer .... m² [2]

A vertical mast stands at *R* and the angle of elevation of the top of the mast from *P* is 18°.

(b) **(i)** Calculate the height of the mast.

$$35 \tan 18°$$
$$= 11.4 \text{ m (3sf)}$$

Answer .... m [2]

(ii) A bird lands on the top of the mast.
Find its largest angle of depression of a point along *PQ* produced.

shortest distance $= 35 \sin 28.0502°$
$$= 16.4585 \text{ m}$$
angle of depression $= \tan^{-1} \dfrac{11.3721}{16.4585}$
$$= 34.6° \text{ (1dp)}$$

Answer .... ° [3]

12

6　(a)　It is given that $\overrightarrow{PQ} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ and the position vector of P is $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Find
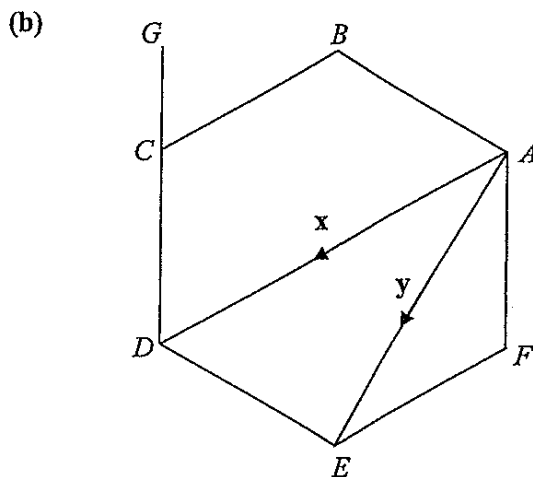
(i)　$\left| \overrightarrow{PQ} \right|$,

7.81

Answer .... units　[1]

(ii)　the coordinates of Q.

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$
$$= \begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} -5 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$Q(-3, 2)$

Answer (............. , ...)　[2]

(b)

$ABCDEF$ is a regular hexagon.
G is a point on DC extended such that $2DG = 3DC$.
$\overrightarrow{AD} = x$ and $\overrightarrow{AE} = y$.

(i)　Express, as simply as possible, in terms of x and/or y,

(a)　$\overrightarrow{AB}$,

$-y + x$

Answer .... [1]

(b) $\overrightarrow{AF}$,

$$y - \frac{1}{2}\,x$$

Answer .... [1]

(c) $\overrightarrow{BG}$,

$$\frac{1}{2}x + \frac{1}{2}\,(\frac{1}{2}x - y)$$
$$= \frac{3}{4}x - \frac{1}{2}\,y$$

Answer .... [2]

(ii) *M* is the mid-point of *AF*.
H is a point on *BE* where $4HE = 3BE$.

(a) Find $\overrightarrow{HG}$.

$$\overrightarrow{HG}$$
$$= \overrightarrow{HB} + \overrightarrow{BG}$$
$$= \frac{1}{2}\,(-y + \frac{1}{2}x) + \frac{3}{4}x - \frac{1}{2}\,y$$
$$= x - y$$

Answer .... [2]

(b) Using vectors, determine whether *H* lies on *MG*.   [2]

$$\overrightarrow{MG}$$
$$= \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BG}$$
$$= \frac{1}{2}\,(-y + \frac{1}{2}x) + x - y + \frac{3}{4}x - \frac{1}{2}\,y$$
$$= 2x - 2y$$
$$= 2(x - y)$$
Since $\overrightarrow{MG} = 2\overrightarrow{HG}$ and *G* is a common point, *H* lies on *BG*.

7 Figure I shows a piece of wood is in the form of a right circular cone with a base diameter of 18 cm.

(a) The curved surface area of the cone is 135π cm^2.
Find the height of the cone.

$\pi(9)l = 135\pi$

$l = 15$

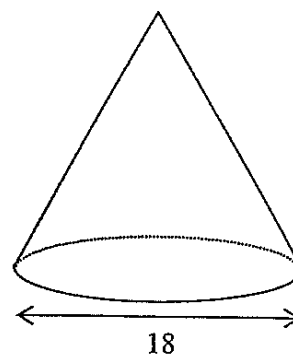height $= \sqrt{15^2 - 9^2}$

$= 12$ cm

Figure I

Answer .... cm [3]

The cone in Figure I is cut into two portions by a plane parallel to its base.
The upper portion is a cone of base diameter 6 cm and the lower portion, as shown in Figure II, is a frustum of height x cm.

(b) Find the value of x.
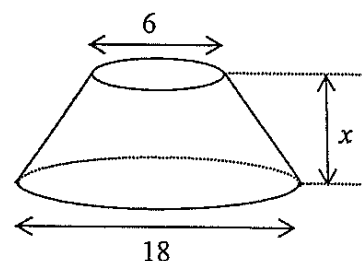
$\dfrac{12-x}{12} = \dfrac{6}{18}$

$12 - x = 4$

$x = 8$

Figure II

Answer $x = $ .... [2]

15

(c) A right cylindrical hole of diameter 6 cm is drilled through the frustum as shown in Figure **III**.

The wood weighs 810 kg per cubic metres.

Find the mass, in grams, of the solid which remains in the frustum.

volume remaining

$$= \frac{1}{3}\pi(9)^2 12 - \frac{1}{3}\pi(3)^2 4 - \pi(3)^2 8$$

$$= 240\pi$$

$$\text{mass} = \frac{810000}{10^6} \times 240\pi$$
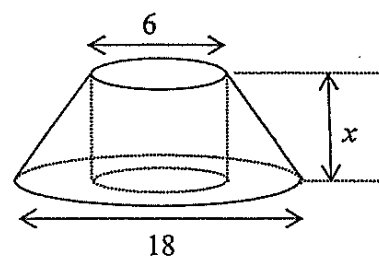
$$= 611 \text{ g (3sf)}$$

Figure **III**

Answer .... g [3]

8   The table shows some values for $y = \frac{3}{10}x^3 - x + 1$ for $-3 \le x \le 3$.

x	-3	-2	-1	0	1	2	3
y	p	0.6	1.7	1	0.3	1.4	6.1

(a)   Find the value of p.
      −4.1

      Answer $p =$ .... [1]

(b)   Using a scale of 2 cm to 1 unit on both axes, draw a horizontal axis for $-3 \le x \le 3$
      and a vertical axis for $-4.5 \le y \le 6.5$ on the grid opposite.
      On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(c)   By drawing a tangent, find the gradient of the curve at $x = 2$.

      Draw tangent and calculate gradient
      2.6 ± 0.2

      Answer   .... [2]

(d)   The equation $\frac{3}{10}x^3 - x = k - 1$, where k is a positive integer, has exactly one solution for
      $-3 \le x \le 3$. Write down an inequality for k.

      $2 \le k \le 6$ or $1 < k < 7$

      Answer   .... [1]

(e)   (i)   The line $y = ax + b$ can be drawn on the same grid to solve the equation
            $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$. Find the value of a and the value of b.

            $$\frac{3}{10}x^3 = \frac{3}{2}x + 1$$

            $$\frac{3}{10}x^3 - x + 1 = \frac{3}{2}x + 1 - x + 1$$

            $$y = \frac{1}{2}x + 2$$

            $$a = \frac{1}{2} \quad \text{and} \quad b = 2$$

            Answer $a =$ .... and $b =$ .... [2]

      (ii) Draw the line $y = ax + b$, for $-3 \le x \le 3$, to solve the equation $\frac{3}{10}x^3 - 1 = \frac{3}{2}x$.

            Draw line for $-3 \le x \le 3$
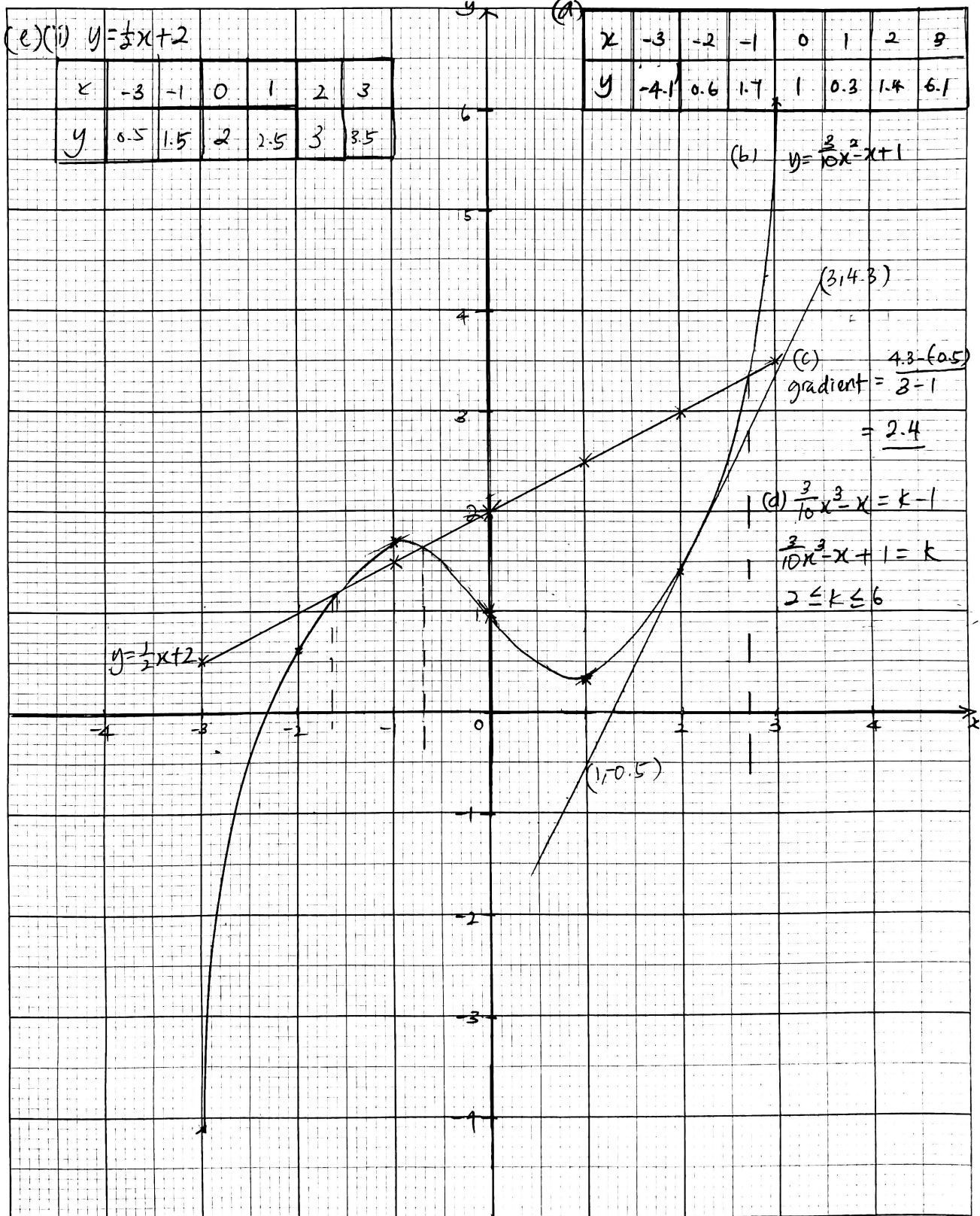 $-1.75, \ -0.75, \ 2.5 \ (\pm 0.05)$

            Answer $x =$ ...., ... or .... [3]

(e)(i) $y = \frac{1}{2}x + 2$

x	-3	-1	0	1	2	3
y	0.5	1.5	2	2.5	3	3.5

(a)

x	-3	-2	-1	0	1	2	3
y	-4.1	0.6	1.7	1	0.3	1.4	6.1

(b) $y = \frac{3}{10}x^2 - x + 1$

$(3, 4.3)$

(c) gradient $= \dfrac{4.3 - (-0.5)}{3 - 1}$

$= \underline{2.4}$

(d) $\frac{3}{10}x^3 - x = k - 1$

$\frac{3}{10}x^3 - x + 1 = k$

$2 \le k \le 6$

$y = \frac{1}{2}x + 2$

$(1, -0.5)$

9   The diagram shows a square *ABCD*.
The diagonals of the square intersect at *W* and *X* is a point on the diagonal *BD*.
Y lies on *CX* such that *DY* is perpendicular to *CX* and *DY* meets *AC* at *Z*.
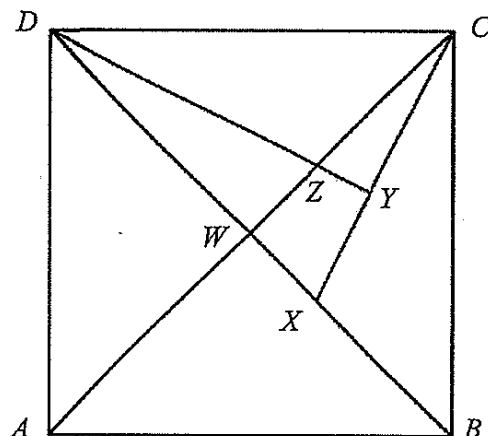
(a)   Show that triangles *DWZ* and *CYZ* are similar.

$\angle DWZ = 90°$ (diagonals of square are perpendicular)

$\angle CYZ = 90°$ (given) $= \angle DWZ$

$\angle DZW = \angle CZY$ (vertically opposite angles)

$\therefore$ Triangles *DWZ* and *CYZ* are similar.

[2]

(b)   Write down another pair of triangles that are similar but not congruent.

$\triangle CXW$ and $\triangle DXY$
OR
$\triangle DAB$ and $\triangle DWA$

Answer $\triangle$.................. and $\triangle$..................   [1]

(c) Prove that triangle *BCX* and triangle *CDZ* have the same area.

$BC = CD$ (sides of square)

$\angle CBX = \angle DCZ$ (diagonal of square bisects interior angle)

$\angle YCZ = \angle WDZ$ ($\triangle DWZ$ and $\triangle CYZ$ are similar)

$\angle BCW = \angle CDW$ (diagonal of square bisects interior angle)

$\angle BCX = \angle BCW - \angle YCZ$

$\qquad = \angle CDW - \angle WDZ$

$\qquad = \angle CDZ$

$\therefore \triangle BCX$ is congruent to $\triangle CDZ$ **(AAS)** and they have the same area.

[4]

(d) It is given that $YZ : WZ = \sqrt{2} : \sqrt{5}$ and $3CZ = 2CW$.

(i) Find $\dfrac{\text{area of } \triangle CYZ}{\text{area of } \triangle DWZ}$.

$\dfrac{2}{5}$

Answer .... [1]

(ii) Calculate the area of the square *ABCD* if the area of triangle *CYZ* is 3 cm².

area of $\triangle DWZ = \dfrac{3}{2} \times 5 = 7.5$

area of square $ABCD = 7.5 \times 12$

$\qquad\qquad\qquad\quad = 90 \text{ cm}^2$

Answer .... cm² [2]

10 John owns a cleaning company that cleans office blocks.
The number of employees in his company and their salaries in 2020 are as follows.

11 handymen, each receiving a yearly salary of $45 936.
272 cleaners, each receiving $5.20 per hour and working 8 hours per day for a total of 20 days per month.
12 supervisors, men and women in the ratio 1 : 3, each receiving a monthy salary that is 235% more than a cleaner's monthy salary.
11 drivers, each receiving $594.50 per week, for 4 weeks in a month.

In addition, both employer and employee must contribute a portion of the employee's monthly salary to the Central Provident Fund (CPF).
The table shows the CPF contribution and allocation rates for Singapore citizens.

Employee's age (years)	Contribution rates (for monthly salary ≥ $750)	
	By employer (% of salary)	By employee (% of salary)
55 and below	17	20
Above 55 to 60	13	13
Above 60 to 65	9	7.5
Above 65	7.5	5

(a) The yearly salary of each handyman in 2020 is 2.5 % more than in 2017.
Find the yearly salary of each handyman in 2017.

$$\frac{45936}{102.5} \times 100$$
$$= \$44815.61$$

Answer $ .... [2]

(b) Mdm Lee is a 65 year old Singapore citizen who works as a cleaner in John's cleaning company. Calculate the total amount of contribution to Mdm Lee's CPF account every month in 2020.

$$\frac{16.5}{100} \times 5.2 \times 8 \times 20$$
$$= \$137.28$$

Answer $ .... [1]

(c) Every month, John randomly selects an 'Employee of the Month' from all his employees. Mr Tan comments that the chance of randomly selecting a male supervisor as the employee of the month is most unlikely.
Do you agree with Mr Tan's comment? Justify your answer. [2]

$$P(\text{male supervisor}) = \frac{3}{11+272+12+11} = \frac{1}{102}$$

$\dfrac{1}{102} = 0.009803$ (3sf) is very close to 0.

Thus, I agree with Mr Tan's claim.

(d) John states that the difference between the salary of the lowest paid employee and the mean salary of all the employees as a percentage of the mean salary of all the employees is less than 25%.
Verify, showing all calculations, whether his statement is valid. [5]

monthly salary of each handyman $= \dfrac{45936}{12} = \$3828$

monthly salary of each cleaner $= 5.2 \times 8 \times 20 = \$832$

monthly salary of each supervisor $= \dfrac{335}{100} \times 832 = \$2787.20$

monthly salary of each driver $= 594.5 \times 4 = \$2378$

mean monthly salary of all employees

$= \dfrac{11(3828) + 272(832) + 12(2787.2) + 11(2378)}{11+272+12+11}$

$= 1071.94902$

Percentage difference $= \dfrac{1071.94902 - 832}{1071.94902} \times 100 = 22.4\%$ (3sf)

22.4% < 25%, John's statement is valid.