Marking Scheme

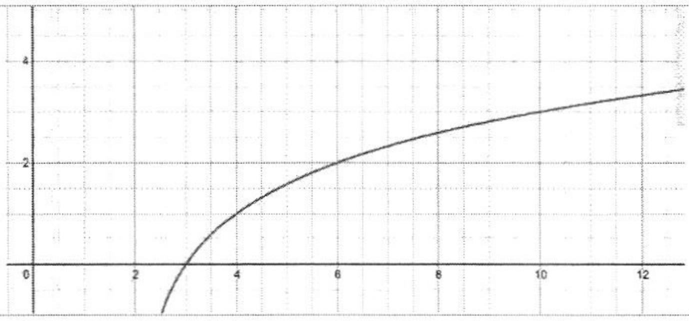
Q	Process	Mark
1	$\left(27+\sqrt{5}\right)=\frac{1}{2}\left(1+\sqrt{5}+3\sqrt{5}-5\right)\times h$	M1
	$\frac{2\left(27+\sqrt{5}\right)}{\left(1+\sqrt{5}+3\sqrt{5}-5\right)}=h$	
	$h=\frac{2\left(27+\sqrt{5}\right)}{\left(-4+4\sqrt{5}\right)}$	
	$=\frac{\left(27+\sqrt{5}\right)\times\left(-2-2\sqrt{5}\right)}{\left(-2+2\sqrt{5}\right)\times\left(-2-2\sqrt{5}\right)}$	M1 (ecf)
	$=\frac{-54-2\sqrt{5}-54\sqrt{5}-10}{4-20}$	
	$=\frac{-64-56\sqrt{5}}{-16}$	M1 (ecf)
	$=\frac{8+7\sqrt{5}}{2}$	A1
		4m
2(a)	$\left(2-3x\right)^8$	
	$=\binom{8}{0}(2)^8(-3x)^0+\binom{8}{1}(2)^7(-3x)^1+\binom{8}{2}(2)^6(-3x)^2+\binom{8}{3}(2)^5(-3x)^3$	M1
	$=256-3072x+16128x^2-48384x^3+\ldots$	A1
2(b)	Sub $x=0.1$	
	1.7^8	
	$=256-3072x+16128x^2-48384x^3+\ldots$	
	$=256-3072(0.1)+16128(0.1)^2-48384(0.1)^3+\ldots$	M1
	$=61.696\,/\,\frac{7712}{125}\,/\,61\frac{87}{125}$	A1
	This is a bad estimation as the above estimation is not very close to the exact value of $1.7^8=69.7575$. (or AS the powers increases, values gets bigger, therefore the values cannot be ignored....)	B1
		5m

Q	Process	Mark
3(i)	$x^2-6x-5=-4x-2$ $x^2-2x-3=0$ $x=-1, x=3$ When $x=-1$, $y=-4(-1)-2$ $=2$ When $x=3$, $y=-4(3)-2$ $=-14$ $(-1,2),(3,-14)$	M1 A1, A1
3(ii)	$x^2-kx-5=-4x-2$ $x^2-kx+4x-3=0$ $D=(-k+4)^2-4(1)(-3)$ $=(-k+4)^2+12$ >0 Since $(-k+4)^2+12>0$, the line intersects the curve at two distinct points.	M1 A1
		5m

Q	Process	Mark
4	$9^x(27)^y = 1 ---(1)$	
	$(\sqrt{2})^{3y} \div (\sqrt{2})^x = 2\sqrt{2} ---(2)$	
	From (1):	
	$3^{2x}(3^{3y}) = 3^0$	
	$2x + 3y = 0 ---(3)$	B1
	From (2):	
	$2^{\frac{3}{2}y} \div 2^{\frac{1}{2}x} = 2^{\frac{3}{2}}$	M1
	$\frac{3}{2}y - \frac{1}{2}x = \frac{3}{2}$	
	$3y - x = 3$	A1
	$-x + 3y = 3 ---(4)$	
	$(3)-(2)$:	
	$(2x+3y)-(-x+3y) = 0-3$	M1 (ecf)
	$3x = -3$	
	$x = -1$	
	When $x = -1$,	
	$-(-1) + 3y = 3$	
	$3y = 2$	
	$y = \frac{2}{3}$	
	$x = -1, y = \frac{2}{3}$	A1, A1
		6m

Q	Process	Mark
5(a)	$f(-2)=6(-2)^3-(-2)^2+2(-2)+3$ $=-53$ Since $f(-2)\neq 0$, so $(x+2)$ is not a factor of $6x^3-x^2+2x+3$.	B1
5(b)	$$\begin{array}{r} 6x-1 \\ x^2-1\overline{)6x^3-x^2+2x+3} \\ -(6x^3+0x^2-6x) \\ \hline -x^2+8x+3 \\ -(-x^2+1) \\ \hline 8x+2 \end{array}$$	M1 (long division)
	$\dfrac{6x^3-x^2+2x+3}{x^2-1}=6x-1+\dfrac{8x+2}{x^2-1}$	A1
	$\dfrac{8x+2}{(x+1)(x-1)}=\dfrac{C}{x-1}+\dfrac{D}{x+1}$ $8x+2=C(x+1)+D(x-1)$	M1
	When $x=1$, $8(1)+2=2C$, $C=5$ When $x=-1$, $8(-1)+2=-2D$, $D=3$	A1
	$\dfrac{6x^3-x^2+2x+3}{x^2-1}=6x-1+\dfrac{5}{x-1}+\dfrac{3}{x+1}$	B1 (ecf)
		6m

Q	Process		Mark
6(a)	$\dfrac{\log_3 5\times\log_5 9}{\log_{49} 7}$	$\dfrac{\log_3 5\times\log_5 9}{\log_{49} 7}$	
	$=\dfrac{\log_3 5\times\dfrac{\log_3 9}{\log_3 5}}{\dfrac{1}{\log_7 49}}$	$=\dfrac{\dfrac{\log_5 5}{\log_5 3}\times\log_5 9}{\dfrac{1}{\log_7 49}}$	M1 (change of base)
	$=\dfrac{\log_3 9}{\frac{1}{2}}$	$=\dfrac{\log_5 9}{\frac{1}{2}}$	M1 (obtain $\frac{1}{2}$ at denominator)
	$=2\log_3 3^2$	$=2\log_3 3^2$	
	$=4$	$=4$	A1
6(b)	$2\log_9 3+\log_5(9y-2)=\log_2 8$		
	$\log_9 3^2+\log_5(9y-2)=\log_2 2^3$		M1 (power law)
	$\log_9 9+\log_5(9y-2)=3$		
	$1+\log_5(9y-2)=3$		
	$\log_5(9y-2)=2$		
	$9y-2=5^2$		M1
	$9y=27$		
	$y=3$		A1
			6m

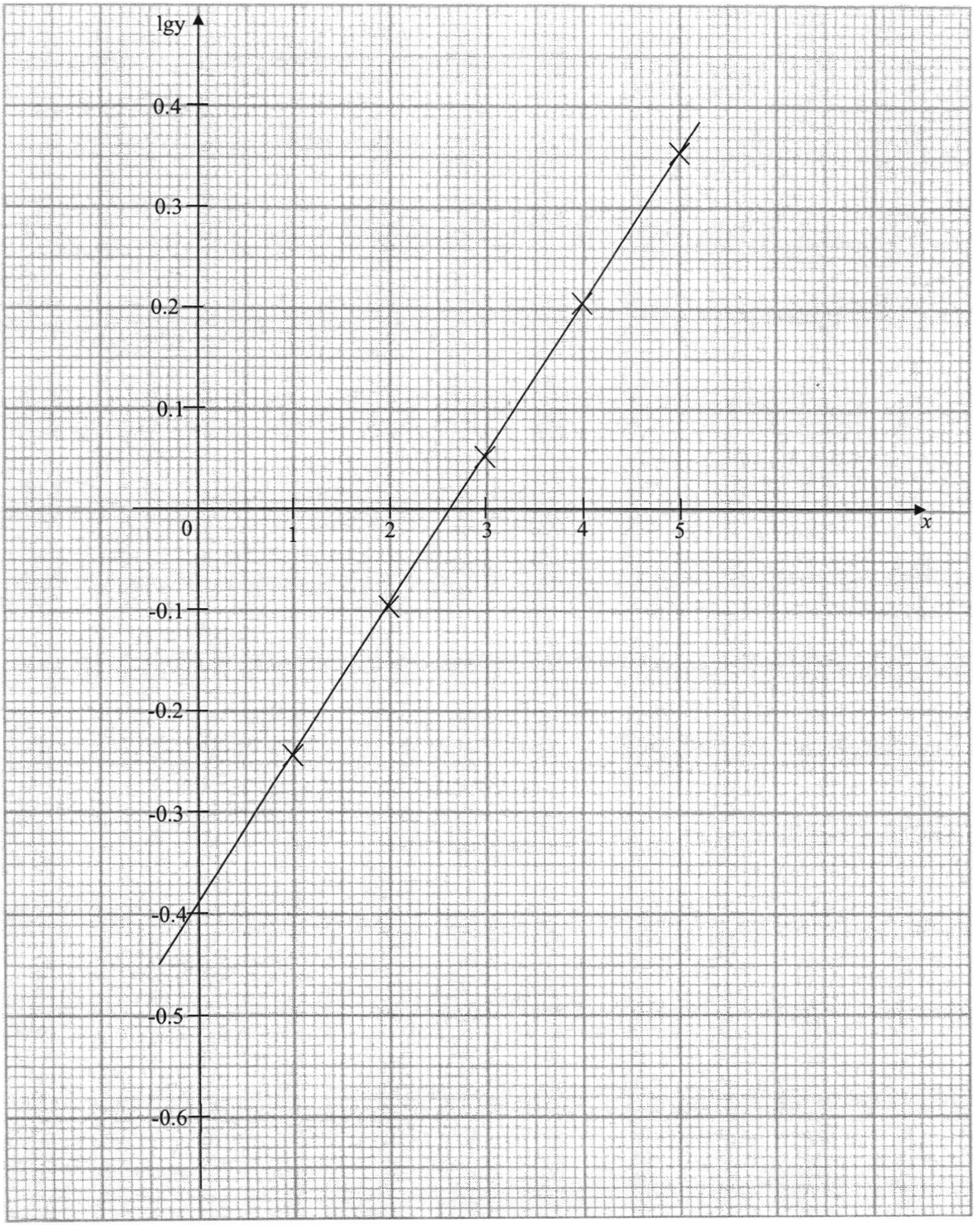
Q	Process	Mark
7(ai)		Shape – B1 x-intercept – B1
7(bi)	When $t = 0$, $P = 8e^{k(0)}$ $P = 8$	B1
7(bii)	When $t = 5$, $16 = 8e^{k(5)}$ $2 = e^{5k}$ $\ln 2 = 5k$ $k = \dfrac{\ln 2}{5}$ $k = 0.1386...$ $= 0.139$	M1 A1
7(biii)	$24 = 8e^{0.1386t}$ $3 = e^{0.1386t}$ $\ln 3 = 0.1386t$ $t = \dfrac{\ln 3}{0.1386}$ $t = 7.9264...$ $= 7.93\text{years}$ $24 = 8e^{\frac{\ln 2}{5}t}$ $3 = e^{\frac{\ln 2}{5}t}$ $\ln 3 = \dfrac{\ln 2}{5}t$ $t = \dfrac{\ln 3}{\frac{\ln 2}{5}}$ $t = 7.92\text{years}$	M1 A1
	Also accept if student round off to 8 years.	
		7m

Q	Process	Mark
8(i)	At $D, x=0$ $y=3(0)+1$ $y=1$ $D(0,1)$	B1
8(ii)	$m_{AD}=3$ $m_{AB}=-\frac{1}{3}$ Eqn of AB: $y-4=-\frac{1}{3}(x-5)$ $y-4=-\frac{1}{3}x+\frac{5}{3}$ $y=-\frac{1}{3}x+\frac{17}{3}$ Sub $y=-\frac{1}{3}x+\frac{17}{3}$ into $y=3x+1$ $-\frac{1}{3}x+\frac{17}{3}=3x+1$ $-\frac{10}{3}x=-\frac{14}{3}$ $x=1.4$ Sub $x=1.4$, $y=3(1.4)+1$ $y=5.2$ $A(1.4,5.2)$	M1 (ecf) M1 M1 (either x or y is correct) (ecf) A1
8(iii)	$C(3,-2)$	B1
8(iv)	Area of Trapezium $=\frac{1}{2}\begin{vmatrix}0 & 1.4 & 5 & 3 & 0\\ 1 & 5.2 & 4 & -2 & 1\end{vmatrix}$ $=\frac{1}{2}\left\|(0+5.6-10+3)-(0+12+26+1.4)\right\|$ $=\frac{1}{2}\left\|-\frac{7}{5}-39.4\right\|$ $=20.4\text{units}^2$ Or $\frac{1}{2}\left(\sqrt{19.6}+\sqrt{40}\right)\times\sqrt{14.4}$ $=20.4unit^2$	M1 A1
		8m

Q	Process	Mark
9(a)	For $a > 0$, $(x+2)(x-7) \le 0$ $x^2 - 7x + 2x - 14 \le 0$ $x^2 - 5x - 14 \le 0$ $\therefore a > 0, p = -5, q = 14$	B1, B1, B1 M1 for $(x+2)(x-7)$ seen
9(bi)	$h = -2t^2 + 3t + 1$ $= -2\left(t^2 - \frac{3}{2}t - \frac{1}{2}\right)$ $= -2\left[\left(t - \frac{3}{4}\right)^2 - \frac{1}{2} - \left(-\frac{3}{4}\right)^2\right]$ or $= -2\left[\left(t - \frac{3}{4}\right)^2 - \frac{17}{16}\right]$ $= -2\left(t - \frac{3}{4}\right)^2 + \frac{17}{8}$ $h = -2t^2 + 3t + 1$ $= -2\left(t^2 - \frac{3}{2}t\right) + 1$ $= -2\left[\left(t - \frac{3}{4}\right)^2 - \left(-\frac{3}{4}\right)^2\right] + 1$ $= -2\left[\left(t - \frac{3}{4}\right)^2 - \frac{9}{16}\right] + 1$ $= -2\left(t - \frac{3}{4}\right)^2 + \frac{17}{8}$	M1 M1 A1
9(bii)	Maximum height $= \frac{17}{8}$ $= 2.125\text{m}$ Since the maximum height the stone can reach is 2.125m which is shorter than 2.6m	 M1 A1
		8m

Q	Process	Mark
10(a)	$(1+hx)^6+(2-x)^7$	
	$=\ldots\binom{6}{3}(1)^3(hx)^3\ldots+\ldots\binom{7}{3}(2)^4(-x)^3\ldots$	M1, M1
	$=\ldots+20h^3x^3+(35)(16)(-x^3)$	
	$=\ldots20h^3x^3-560x^3\ldots$ $\therefore 20h^3-560=720$	M1(ecf)
	$h^3=64$	
	$h=4$	A1
10(bi)	$T_{r+1}=\binom{6}{r}(x)^{6-r}\left(\frac{4}{x^2}\right)^r$ or $T_{r+1}=\binom{6}{r}(x)^{6-3r}(4)^r$	B1
10(bii)	To find the value r for independent term, $6-r-2r=0$	M1
	$-3r=-6$	
	$r=2$	A1
	Independent term $=\binom{6}{2}(4)^2$	
	$=240$	B1
		8m

Q	Process						Mark
11(i)	x	1	2	3	4	5	Points – B1 Line – B1
	y	0.566	0.80	1.13	1.60	2.26	
	lgy	-0.2472	-0.0969	0.0530	0.204	0.3541	



Q	Process	Mark
11(ii)	$my = n(2^{mx})$ $y = \frac{n}{m}(2^{mx})$ $\lg y = \lg\left(\frac{n}{m}\right) + (mx)\lg 2$ $\lg y = \lg\left(\frac{n}{m}\right) + m\lg 2(x)$ $\text{gradient} = m\lg 2$ $\frac{0.3-(-0.3)}{4.1-0.6} = m\lg 2$ $\frac{0.6}{3.5} \div (\lg 2) = m$ $m = 0.48613...$ $m = 0.486$ $\text{y-intercept} = \lg\left(\frac{n}{m}\right)$ $-0.39 = \lg\left(\frac{n}{0.48613}\right)$ $n = 0.19803...$ $n = 0.198$	 M1 M1 A1 (accept $m =$ 0.486 to 0.631) A1 (accept $n =$ 0.198 to 0.263)
11(iii)	$x = 2.3$ $\lg y = -0.05$ $y = 10^{-0.05}$ $= 0.891$	B1 (value for lg y) B1 ($10^{\text{their ans}}$) B1 (accept 0.871 to 0.912)
		9m

Q	Process	Mark
12(i)	$\mathrm{f}(x) = x^3 + hx^2 + kx - 3$	
	$\mathrm{f}(3) = 0$	
	$(3)^3 + h(3)^2 + k(3) - 3 = 0$	M1
	$27 + 9h + 3k - 3 = 0$	
	$9h + 3k = -24$	
	$3h + k = -8$	A1
	$\mathrm{f}(-1) = -4$	
	$(-1)^3 + h(-1)^2 + k(-1) - 3 = -4$	M1
	$-1 + h - k - 3 = -4$	
	$h - k = 0$	
	$h = k$	A1
	$\therefore 4h = -8$	
	$h = -2$	
	$k = -2$	A1
12(ii)	$\mathrm{f}(x) = x^3 - 2x^2 - 2x - 3$	
	$\mathrm{f}(x) = (x-3)(x^2 + bx + 1)$	M1
	By comparing coef of x^2,	
	$-3 + b = -2$	
	$b = 1$	
	$\mathrm{f}(x) = (x-3)(x^2 + x + 1)$	M1
	$y = x^2 + x + 1$	
	$D = (1)^2 - 4(1)(1)$	
	$= -3$	
	< 0	M1 (ecf)
	Since $D < 0$, there is only one solution for $\mathrm{f}(x)$.	A1
		9m

Q	Process	Mark
13(a)	The value of the radius is 5. The y coordinate of the centre of the circle is 5.	B1 B1
13(bi)	Let the x coordinate of the centre be a. Let the centre be $(a, 5)$ Equation of circle: $(x-a)^2+(y-5)^2=25$ *When* $x=-7$, $y=9$: $(-7-a)^2+4^2=25$ $(-7)^2+14a+a^2=9$ $a^2+14a+40=0$ $a=-4$ *or* $a=-10$ (rej) Equation of circle: $(x+4)^2+(y-5)^2=25$ *or* $x^2+8x+y^2-10y+16=0$	M1 M1(ecf) A1 B1 (ecf)
13(bii)	Let the centre of the circle be A. $m_{AP}=\dfrac{9-5}{-7+4}$ $=-\dfrac{4}{3}$ $m_{\tan gent}=\dfrac{3}{4}$ $y=\dfrac{3}{4}x+c$ $9=\dfrac{3}{4}(-7)+c$ $c=\dfrac{57}{4}$ $Equation_{\tan gent}: y=\dfrac{3}{4}x+\dfrac{57}{4}$ *or* $y=\dfrac{3}{4}x+14\dfrac{1}{4}$	M1 (ecf) M1 (ecf) A1
		9m