FSS 2021 3Express EOY

Additional Mathematics (90 marks)

Qn. #	Solution	Mark Allocation
1a	$2x^2-4x+7$ $=2\left(x^2-2x+\frac{7}{2}\right)$ $=2\left[x^2-2x+\left(-\frac{2}{2}\right)^2-\left(-\frac{2}{2}\right)^2+\frac{7}{2}\right]$ $=2\left[(x-1)^2+\frac{5}{2}\right]$ $=2(x-1)^2+5$	M1 (take out factor 2) A1 ($a(x+b)^2+c$ form)
	$-x^2-4x-1$ $=-(x^2+4x+1)$ $=-\left[x^2+4x+\left(\frac{4}{2}\right)^2-\left(\frac{4}{2}\right)^2+1\right]$ $=-\left[(x+2)^2-3\right]$ $=-(x-1)^2+3$	M1 (take out factor -1) A1 ($a(x+b)^2+c$ form)
1b	Minimum value of $2x^2-4x+7=5$ Maximum value of $-x^2-4x-1=3$ Since the minimum value of $y=2x^2-4x+7$ is higher than the maximum value of $y=-x^2-4x-1$, they will not intersect.	B1 (min value) B1 (max value) B1 (conclusion)

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2a	$4x-7\sqrt{3}=x\sqrt{3}+5$ $4x-x\sqrt{3}=5+7\sqrt{3}$ $x\left(4-\sqrt{3}\right)=5+7\sqrt{3}$ $x=\dfrac{5+7\sqrt{3}}{4-\sqrt{3}}\left(\times\dfrac{4+\sqrt{3}}{4+\sqrt{3}}\right)$ $=\dfrac{20+5\sqrt{3}+28\sqrt{3}+21}{16-3}$ $=\dfrac{41+33\sqrt{3}}{13}$ $=\dfrac{41}{13}+\dfrac{33}{13}\sqrt{3}$	M1 (factorise; $\left(4-\sqrt{3}\right)$ seen) M1 (rationalise surds) A1 (in the form $a+b\sqrt{3}$)
2b	$2\sqrt{x-3}+x=11$ $2\sqrt{x-3}=11-x$ $4(x-3)=(11-x)^2$ $4x-12=121-22x+x^2$ $x^2-26x+133=0$ $(x-7)(x-19)=0$ $x=7$ or $x=19$ (rejected)	M1 (square both sides) M1 (factorise) A1 (reject $x=19$)
3a	$2x^2+bx+17-7x-9=0$ $2x^2+(b-7)x+8=0$ Since curve and line intersect at two distinct points, $b^2-4ac>0$ $(b-7)^2-4(2)(8)>0$ $b^2-14b+49-64>0$ $b^2-14b-15>0$ $(b-15)(b+1)>0$ $\therefore b<-1$ or $b>15$	M1 (substitution method) M1 (correct discriminant) M1 (factorise) A1

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3b	$x^2+k(2x+1)+12=x^2+2kx+k+12$ Since curve is always positive, $b^2-4ac<0$ $(2k)^2-4(1)(k+12)<0$ $4k^2-4k-48<0$ $k^2-k-12<0$ $(k-4)(k+3)<0$ $\therefore -3<k<4$	M1 (correct discriminant) M1 (factorise) A1
4a	Let $f(x)=x^2+7x+3$ $f(p)=(p)^2+7(p)+3$ $f(p)=p^2+7p+3$ - (1) $f(-q)=(-q)^2+7(-q)+3$ $f(-q)=q^2-7q+3$ - (2) (1) = (2) $p^2+7p+3=q^2-7q+3$ $p^2-q^2+7p+7q=0$ $(p+q)(p-q)+7(p+q)=0$ $(p+q)(p-q+7)=0$ $p+q=0$ or $p-q+7=0$ (reject) $p-q=-7$	M1 (either equation seen) M1 (substitution method) M1 (factorise) A1 (no mark award if student did not reject $p+q=0$)

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4b	$f(-2)=2(-2)^3+(-2)^2-10(-2)-8=0$ Therefore, $(x+2)$ is a factor of $f(x)=2x^3+x^2-10x-8$. (shown)	B1
	$2x^3+x^2-10x-8=(x+2)(ax^2+bx+c)$ Comparing coefficient of x^3, $a=2$ Comparing constant, $2c=-8$ $c=-4$ Comparing coefficient of x, $c+2b=-10$ $-4+2b=-10$ $b=-3$	
	$2x^3+x^2-10x-8=(x+2)(2x^2-3x-4)$	M1 (compare coefficients / long division)
	$2x^3+x^2-10x-8=0$ $(x+2)(2x^2-3x-4)=0$	A1 ($2x^2-3x-4$ seen)
	$x=-2$ or $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)}$	M1 (solve quadratic equation)
	Therefore $x=-2$, $x=2.35$, or $x=-0.851$.	A1

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5	By long division, $\frac{x^3+2x^2-6}{x^2-9}=x+2+\frac{9x+12}{x^2-9}$ $\frac{9x+12}{x^2-9}=\frac{9x+12}{(x-3)(x+3)}=\frac{A}{x-3}+\frac{B}{x+3}$ $9x+12=A(x+3)+B(x-3)$ When $x=3$, $9(3)+12=A(3+3)$ $A=\frac{13}{2}$ When $x=-3$, $9(-3)+12=B(-3-3)$ $B=\frac{5}{2}$ $\frac{9x+24}{x^2-9}=\frac{13}{2(x-3)}+\frac{5}{2(x+3)}$ $\frac{x^3+2x^2-6}{x^2-9}=x+2+\frac{13}{2(x-3)}+\frac{5}{2(x+3)}$	M1 (long division) M1 (factorise $x^2-9=(x-3)(x+3)$) M1 (A found, ecf) M1 (B found, ecf) A1
6a	$(a+x)^4+\left(2-\frac{x}{4}\right)^5$ $=a^4+\binom{4}{1}a^3x+\binom{4}{2}a^2x^2+\ldots+2^5+\binom{5}{1}2^4\left(-\frac{x}{4}\right)+\binom{5}{2}2^3\left(-\frac{x}{4}\right)^2+\ldots$ $=a^4+4a^3x+6a^2x^2+80-20x+5x^2+\ldots$ Comparing coefficient of x^2, $6a^2+5=29$ $6a^2=24$ $a^2=4$ $a=2$ (reject $a=-2$)	M1 $a^4+\binom{4}{1}a^3x+\binom{4}{2}a^2x^2+\ldots$ seen M1 $2^5+\binom{5}{1}2^4\left(-\frac{x}{4}\right)+\binom{5}{2}2^3\left(-\frac{x}{4}\right)^2+\ldots$ seen M1 A1 (rejected $a=-2$)
6b	Coefficient of x $=4a^3-20$ $=4(2)^3-20$ $=12$	M1 A1

Qn. #	Solution	Mark Allocation
7a	$\log_x 4y = \dfrac{\log_2 4y}{\log_2 x}$ $= \dfrac{\log_2 4 + \log_2 y}{\log_2 x}$ $= \dfrac{2+q}{p}$	M1 (change base to 2) M1 (product law) A1
7b	$2\log_2 x - \log_2 (x-5) = 2$ $\log_2 \dfrac{x^2}{(x-5)} = 2$ $\dfrac{x^2}{(x-5)} = 2^2$ $x^2 = 4(x-5)$ $x^2 - 4x + 20 = 0$ Discriminant $= b^2 - 4ac$ $= (-4)^2 - 4(1)(20)$ $= -64 < 0$ Since discriminant < 0, the equation has no real solutions.	M1 (quotient law) M1 (change log to exponential) M1 (form quadratic equation) M1 (find value of discriminant / used quadratic formula) A1 (show that when discriminant < 0, there are no real solutions / concluded that no answer from quadratic formula)
8a	$N = \dfrac{15000}{1+9999e^{-0.2(7)}} = 6.080942101 = 6$ 6 residents are likely to have contracted influenza by Day 7.	B1
8b	As $t \to \infty$, $9999e^{-0.2t} \to 0$ $\dfrac{15000}{1+9999e^{-0.2t}} \to 15000$ The population of residents in Hugetown is only 15 000.	B1

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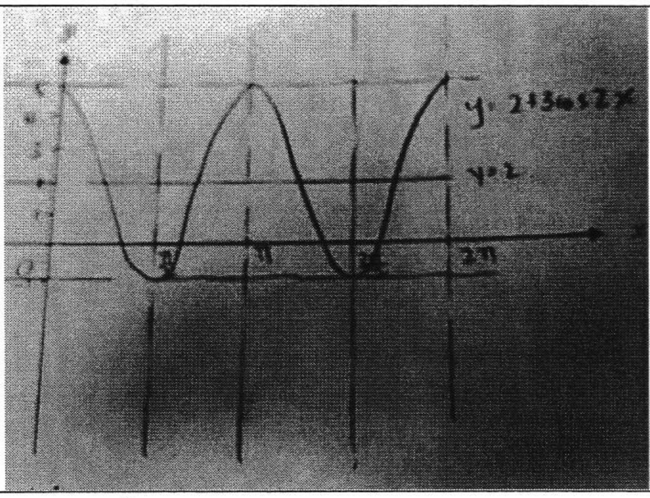
8c	$\dfrac{15000}{1+9999e^{-0.2t}}=3000$	M1 (3000 seen)
	$1+9999e^{-0.2t}=5$	
	$e^{-0.2t}=\dfrac{4}{9999}$	
	$-0.2t=\ln\dfrac{4}{9999}$	M1 (take ln)
	$t=39.1197300293$	A1
	$\therefore$Day 40	
9	$2^{3y}=2^{-2x}$ $3^{3y}\div 3^{\frac{1}{2}x}=3^{4}\left(3^{\frac{1}{2}}\right)$	M1 (law of indices)
	$3y=-2x$ $3^{3y-\frac{1}{2}x}=3^{4+\frac{1}{2}}$	
	$y=-\dfrac{2}{3}x$ - (1) $3y-\dfrac{1}{2}x=\dfrac{9}{2}$ - (2)	M1 (both equations)
	Sub (1) into (2),	
	$3\left(-\dfrac{2}{3}x\right)-\dfrac{1}{2}x=\dfrac{9}{2}$	M1 (substitution method)
	$-\dfrac{5}{2}x=\dfrac{9}{2}$	
	$x=-\dfrac{9}{5}$	A1
	Sub $x=-\dfrac{9}{5}$ into (1),	
	$y=-\dfrac{2}{3}\left(-\dfrac{9}{5}\right)$	
	$y=\dfrac{6}{5}$	A1
	Therefore, $x=-\dfrac{9}{5}$ and $y=\dfrac{6}{5}$.	

Qn. #	Solution	Mark Allocation
10a	$x^2+y^2+2gx+2fy+c=0$, $r=\sqrt{g^2+f^2-c}$ Given $x^2+y^2-27x+41=0$, Comparing coefficient of x, $2g=-27$ $g=-\frac{27}{2}$ Comparing coefficient of y, $-2f=0$ $f=0$ Comparing constant, $c=41$ Centre $\left(\frac{27}{2},0\right)$ and $r=\sqrt{\left(-\frac{27}{2}\right)^2-41}$ $=11.9$ units^2 (3 s.f.)	M1 (either g, f or c found) A1 (centre) M1 (radius) A1
10b	Let the equation of the straight line be $y=mx+c$. Line passes through $(0,-3)$ and $P(2,3)$. $m=\frac{3+3}{2-0}$ $=3$ $y=3x-3$ $x^2+y^2-27x+41=0$ - (1) $y=3x-3$ - (2) Sub (2) into (1). $x^2+(3x-3)^2-27x+41=0$ $x^2+9x^2-18x+9-27x+41=0$ $10x^2-45x+50=0$ $2x^2-9x+10=0$ $(2x-5)(x-2)=0$ $x=\frac{5}{2}$ or $x=2$ (reject; x-coordinate of P) $y=\frac{9}{2}$ Therefore, $Q\left(\frac{5}{2},\frac{9}{2}\right)$	M1 (equation of line) M1 (substitution method) M1 (quadratic equation form) A1 (coordinates of Q found)

Qn. #	Solution	Mark Allocation
11a	Let the coordinates of $C(x,y)$. E is the midpoint of AC. $\left(\frac{x+6}{2},\frac{y+6}{2}\right) = (4,2)$ $\frac{x+6}{2}=4$ or $\frac{y+6}{2}=2$ $x=2$ $y=-2$ Therefore, $C(2,-2)$.	M1 (Midpoint) A1
11b	Let the equation of AB be $y=m_{AB}+c$. $m_{OC} = \frac{-2-0}{2-0}$ $=-1$ $m_{AB}=m_{OC}=-1$ Sub $A(6,6)$ and $m_{AB}=-1$, $y-6=-(x-6)$ $y=-x+12.$ Let the equation of BD be $y=m_{BD}+c$. $m_{AC} = \frac{6+2}{6-2}$ $=2$ $m_{BD}=-\frac{1}{m_{AC}}=-\frac{1}{2}$ E lies on line BD $y-2=-\frac{1}{2}(x-4)$ $y=-\frac{1}{2}x+4.$ B lies on AB and BD, $y=-x+12.$ - (1) $y=-\frac{1}{2}x+4.$ - (2) $(1)=(2)$,	 M1 (parallel gradient) M1 (equation of AB) M1 (perpendicular gradient) M1 (equation of BD)

Qn. #	Solution	Mark Allocation
	$-x+12=-\frac{1}{2}x+4$ $-\frac{1}{2}x=-8$ $x=16$ $y=-16+12=-4$ $B(16,-4)$. *D* lies on *y* axis and *BD*, When $x=0,\ y=-\frac{1}{2}(0)+4=4$ $D(0,4)$.	M1 (substitution) A1 (*B*) A1 (*D*)
11c	Area of *ABCD* $=\frac{1}{2}\begin{vmatrix} 0 & 2 & 16 & 6 & 0 \\ 4 & -2 & -4 & 6 & 4 \end{vmatrix}$ $=\frac{1}{2}\lvert 112-(-48)\rvert$ $=80\text{ units}^2$.	M1 (Simplification) A1
12a	Period $=\frac{2\pi}{2}$ $=\pi$ amplitude $=3$	B1 B1
12b		B1 correct shape and correct max/min value B1 correct number of cycles
13a	$\cos A=\frac{5}{13}$	B1
13b	$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $=\frac{12}{13}\left(\frac{4}{5}\right)+\frac{5}{13}\left(\frac{3}{5}\right)$ $=\frac{63}{65}$	M1 A1

Qn. #	Solution	Mark Allocation
14a	$\text{LHS}=\dfrac{\sec x-\operatorname{cosec} x}{\sec x+\operatorname{cosec} x}$ $=\dfrac{\dfrac{1}{\cos x}-\dfrac{1}{\sin x}}{\dfrac{1}{\cos x}+\dfrac{1}{\sin x}}$ $=\dfrac{\dfrac{\sin x-\cos x}{\sin x\cos x}}{\dfrac{\sin x+\cos x}{\sin x\cos x}}$ $=\dfrac{\dfrac{\sin x-\cos x}{\cos x}}{\dfrac{\sin x+\cos x}{\cos x}}$ $=\dfrac{\tan x-1}{\tan x+1}$	M1 (reciprocal for either sec x or cosec x) M1 (common denominator) A1 (shown)
14b	$\dfrac{\tan x-1}{\tan x+1}=\dfrac{4}{9}$ $9\tan x-9=4\tan x+4$ $5\tan x=13$ $\tan x=\dfrac{13}{5}$ $\alpha=1.2036...$ $x=1.2036...,\ \pi+1.2036...$ $x\approx 1.20\ ,\ 4.35$	M1 M1 (basic angle) A1 (both values)