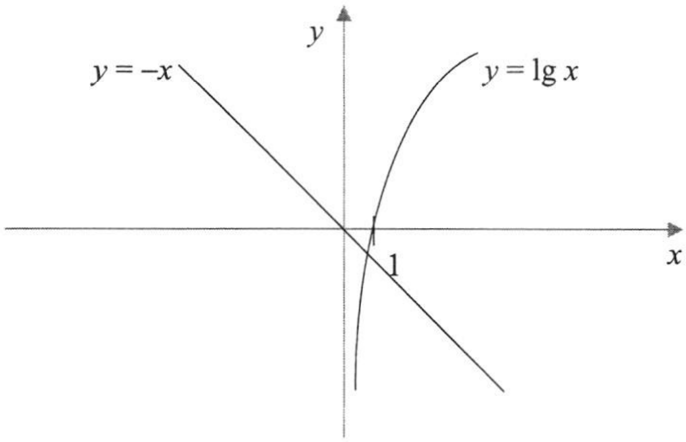
Beatty Secondary School
EOY 2021
3E Additional Mathematics

Qn	Solution
1(i)	A and B are in the 2[nd] quadrant. For angle A, adj $= -\sqrt{5^2-3^2} = -4$ ... **[M1] Accept "+4"** $\tan A = -\frac{3}{4}$ ... **[A1]**
1(ii)	For angle B, opp $= \sqrt{25^2-7^2} = 24$ ... **[M1]** $\text{cosec } B = \frac{1}{\sin B} = \frac{25}{24}$ ... **[A1]**
2.	$2^{x+1} + 2^{x+2} = 6^x$ $2^x(2^1+2^2) = 6^x$ ... **[M1]** $2^x(6) = 6^x$ $\frac{6^x}{2^x} = 6$ $3^x = 6$ ... **[A1]** $x = \frac{\ln 6}{\ln 3} = 1.631$ (to 4sf) ... **[A1]**
3(i)	$BC = \sqrt{\left(3-\sqrt{5}\right)^2 + \left(3+\sqrt{5}\right)^2 - 2\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)\cos 60°}$ ... **[M1]** $= \sqrt{9-6\sqrt{5}+5+9+6\sqrt{5}+5-2(9-5)\left(\frac{1}{2}\right)}$ ... **[M1]** $= \sqrt{24}$ $= 2\sqrt{6}$ m ... **[A1]**
(ii)	$\frac{1}{2}\left(\ +\sqrt{5}\right)\left(\ -\sqrt{\ }\right) \qquad \circ = \frac{1}{2}(h)\left(\ \sqrt{\ }\right)$ ... **[M1]** $(9-5)\left(\frac{\sqrt{3}}{2}\right) = \left(2\sqrt{6}\right)h$ $2\sqrt{3} = \left(2\sqrt{6}\right)h$ ... **[M1]**

	$h=\frac{\sqrt{3}}{\sqrt{6}}$ $=\frac{1}{\sqrt{2}}$ $=\frac{1}{2}\sqrt{2}$ m … **[A1]**
4.	$3y = 6 - 2x$ $x = 3 - 1.5y$ … (1) Sub (1) into $x^2 - y^2 = 9$ $(3-1.5y)^2 - y^2 = 9$ … **[M1]** $9-9y+2.25y^2-y^2=9$ $1.25y^2-9y=0$ $5y^2-36y=0$ $y(5y-36)=0$ … **[M1]** $y = 0$ or $y = 7.2$ … **[A1]** When $y = 0$, $x = 3 - 1.5(0) = 3$ When $y = 7.2$, $x = 3 - 1.5(7.2) = -7.8$ } …**[A1]** Midpoint of $AB = \left(\frac{3-7.8}{2}, \frac{0+7.2}{2}\right) = (-2.4, 3.6)$ … **[A1]**
5(i)	$2x^2+12x+9$ $=2(x^2+6x)+9$ $=2\left[(x+3)^2-9\right]+9$ … **[M1]** $=2(x+3)^2-9$ … **[M1]** Minimum point = (–3, –9) … **[A1]** -- *Alternatively,* *Let* $2x^2+12x+9=0$. $x=\frac{-12\pm\sqrt{72}}{4}$ … ***[M1]*** *x-coordinate of min pt* = $\left(\frac{-12+\sqrt{72}}{4}+\frac{-12-\sqrt{72}}{4}\right)\div 2 = -3$ … ***[M1]*** *y-coordinate of min pt* = $2(-3)^2+12(-3)+9=-9$ *Minimum point = (–3, –9)* … ***[A1]***

(ii)	$2x^2+12x+9=mx+1$ … **[M1]** $2x^2+(12-m)x+8=0$ Discriminant > 0 $(12-m)^2-4(2)(8)>0$ … **[M1]** $(12-m)^2>64$ $m^2-24m+80>0$ $(m-4)(m-20)>0$ … **[M1]** $m<4$ or $m>20$ … **[A1]**
(iii)	Any quadratic equation in the form $y=a(x+3)^2-9$, where $a\neq 2$ or $y=2x^2+bx+c$, where $b\neq 12$ or $y=ax^2+12x+9$, where $a\neq 2$ or any equation where when it is substituted into original equation, the resulting equation is a perfect square ... **[B1]**
6	By long division, $\dfrac{x^2}{x^2-6x+9}=1+\dfrac{6x-9}{x^2-6x+9}$ … **[M1]** $=1+\dfrac{6x-9}{(x-3)^2}$ Let $\dfrac{6x-9}{(x-3)^2}=\dfrac{A}{x-3}+\dfrac{B}{(x-3)^2}$ $6x-9=A(x-3)+B$ … **[M1]** Comparing coefficient of x, $A=6$ … **[A1]** Let $x=3$ $B=6(3)-9=9$ … **[A1]** $\dfrac{x^2}{x^2-6x+9}=1+\dfrac{6}{x-3}+\dfrac{9}{(x-3)^2}$
7	$\log_2 x-3\log_x 2=2$ $\log_2 x-\dfrac{3}{\log_2 x}=2$ … **[M1]** Let $y=\log_2 x$ $y-\dfrac{3}{y}=2$

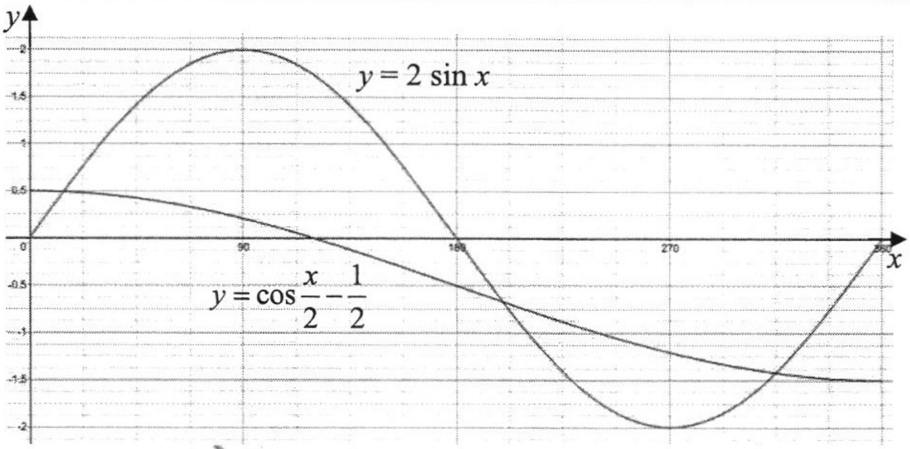
	$y^2 - 2y - 3 = 0$ $(y-3)(y+1) = 0$ **... [M1]** $y = 3$ or $y = -1$ **... [A1]** $\log_2 x = 3$ or $\log_2 x = -1$ $x = 2^3$ or $x = 2^{-1}$ $x = 8$ or $x = 0.5$ **... [A1]**
8(i)	 **... [G1], [G1]**
8(ii)	$\lg x = -x$ **... [M1]** $10^{-x} = x$ $x10^x = 1$ **... [A1]** Since the graphs cut at only one point, there is one solution to $x10^x = 1$. **[A1]** -- *Alternatively,* $x10^x = 1$ $\lg x10^x = \lg 1$ ***... [M1]*** $\lg x + \lg 10^x = 0$ $\lg x + x = 0$ $\lg x = -x$ ***... [A1]*** *Since the graphs cut at only one point, there is one solution to* $x10^x = 1$. ***[A1]***

9(i)	$(2-4x)^7 = 2^7(1-2x)^7$ **… [M1]** $= 128(1-2x)^7$ Since the coefficient of every term in $(1-2x)^7$ is an integer, the coefficient of every term in $128(1-2x)^7$ is a multiple of 128. **… [A1]** ***Accept also if students expand all 8 terms correctly and claim that every coefficient is multiple of 128.*** ***All 8 terms:*** $128-1792x+10752x^2-35840x^3+71680x^4$ $-86016x^5+57344x^6-16384x^7$
9(ii)	$(2-4x)^7$ $= 2^7 - \binom{7}{1}(2^6)(4x) + \binom{7}{2}(2^5)(4x)^2 - \ldots$ **… [M1]** $= 128-1792x+10752x^2-\ldots$ **… [A1]**
9(iii)	$(1+kx)(2-4x)^7$ $= (1+kx)(128-1792x+10752x^2-\ldots)$ Coefficient of $x^2 = 10752 - 1792k$ **… [M1]** $10752 - 1792k = 0$ **… [M1]** $k = 6$ **… [A1]**
10(i)	Let $f(x) = x^3+ax+b$ $f(-4) = 0$ and $f(1) = -5$ $(-4)^3 + a(-4) + b = 0$ **… [M1]** $-64-4a+b=0$ … (1) $1^3 + a(1) + b = -5$ **… [M1]** $6+a+b=0$ … (2) (2) – (1) $5a + 70 = 0$ $a = -14$ **… [A1]** When $a = -14$, $6 - 14 + b = 0$ $b = 8$ **… [A1]**

(ii)	$x^3 - 14x + 8 = (x+4)(x^2 + kx + 2)$ Comparing coefficient of x, $-14 = 2 + 4k$ $k = -4$ … **[M1] (Accept long division too)** $x^3 - 14x + 8 = (x+4)(x^2 - 4x + 2) = 0$ $x = -4$ … **[A1]** or $x = \frac{4 \pm \sqrt{4^2 - 4(1)(2)}}{2}$ … **[M1]** $= \frac{4 \pm \sqrt{8}}{2}$ $= \frac{4 \pm 2\sqrt{2}}{2}$ $= 2 \pm \sqrt{2}$ … **[A1]**
11	At $(x, y) = (2, 1.75)$ and $(-1, 1)$, $(\frac{1}{x}, xy) = (0.5, 3.5)$ and $(-1, -1)$ … **[M1]** Gradient $= \frac{3.5+1}{0.5+1}$ … **[M1]** $= 3$ $xy + 1 = 3\left(\frac{1}{x} + 1\right)$ … **[M1]** $xy = \frac{3}{x} + 2$ When $x = 3$, $3y = 1 + 2$ … **[M1]** $y = 1$ … **[A1]**
12(i)	$P = 55e^{kt} + 5$ To get initial population, let $t = 0$ ⇨ $P = 55e^0 + 5$ … **[M1]** $= 60$ Let $P = 120$ and $t = 2$ $120 = 55e^{2k} + 5$ $55e^{2k} = 115$ $e^{2k} = \frac{23}{11}$ … **[M1]**

	$k=\dfrac{\ln\frac{23}{11}}{2}$ ≈ 0.36879 $= 0.369$ (to 3sf) **... [A1]**
12(ii)	$1000 = 55e^{0.36879t} + 5$ $e^{0.36879t} = \dfrac{199}{11}$ $t=\dfrac{\ln\frac{199}{11}}{0.36879}$ **... [M1]** $= 7.85$ (3sf) Greatest integer $t = 7$ **... [A1]**
13(i)	Gradient of normal = $-\dfrac{4}{3}$ **... [M1]** $y-7=-\dfrac{4}{3}(x+3)$ **... [M1]** $y=-\dfrac{4}{3}x+3$ **... [A1]**
13(ii)	Gradient of AB = $\dfrac{5-7}{-5+3}=1$ Gradient of perpendicular bisector of $AB = -1$ **... [M1]** Midpoint of $AB = \left(\dfrac{-3-5}{2},\dfrac{7+5}{2}\right) = (-4, 6)$ **... [M1]** $y-6=-(x+4)$ $y = -x + 2$ ... (1) **... [M1]** --- *Alternatively,* $AC = BC$ $\sqrt{\qquad y-7)^2} = \sqrt{\qquad y-5)^2}$ ***... [M1]*** $x^2+6x+9+y^2-14y+49 = x^2+10x+25+y^2-10y+25$ ***... [M1]*** $4y + 4x = 8$ $y = -x + 2$ ... (1) ***... [M1]***

	Sub (1) into $y=-\frac{4}{3}x+3$ $-x+2=-\frac{4}{3}x+3$ … **[M1]** $\frac{1}{3}x=1$ $x=3$ When $x = 3$, $y = -3 + 2 = -1$ Hence, $C = (3, -1)$ (shown) … **[A1]**
13(iii)	Radius of circle = $\sqrt{(3+3)^2+(-1-7)^2}=10$ … **[M1]** Equation of circle is $(x-3)^2+(y+1)^2=100$ … **[A1]** *Accept also* $x^2+y^2-6x+2y-90=0$

14(i)	 **G1** for correct amplitude for both graphs **G2** for correct number of cycles for both graphs **G1** for correct curves (including labelling of axes)
14(ii)	There is no point of intersection … **[B1]** between the two graphs where x is between 90° and 180° …**[B1]**
15(i)	Let $B = (b, 0)$ and $D = (0, d)$

$\tan \angle DBO = 2$

$\dfrac{d-0}{0-b} = 2$ **… [M1]**

$d = -2b$ … (1)

$BC = CD$

$\sqrt{b \qquad\qquad -6)^2} = \sqrt{\qquad\qquad\qquad d-6)^2}$ **… [M1]**

$b^2 + 12b + 36 + 36 = 36 + d^2 - 12d + 36$

$b^2 + 12b = d^2 - 12d$ … (2)

Sub (1) into (2)

$b^2 + 12b = (-2b)^2 - 12(-2b)$ **… [M1]**

$b^2 + 12b - 4b^2 - 24b = 0$

$-3b^2 - 12b = 0$

$-3b(b+4) = 0$ **… [M1]**

$b = 0$ or $b = -4$

(Rej)

When $b = -4$, $d = -2(-4) = 8$

Hence, $B = (-4, 0)$ and $D = (0, 8)$ (shown) **… [A1]**

15(ii)

Gradient of $BD = \dfrac{8-0}{0+4} = 2$ **… [M1]**

Gradient of $AC = -\dfrac{1}{2}$

Let $A = (p, q)$

$\dfrac{q-6}{p+6} = -\dfrac{1}{2}$ **… [M1]**

$2(q-6) = -p-6$

$p = 6 - 2q$ … (3)

Area = 80

$\dfrac{1}{2}\begin{vmatrix} -6 & -4 & 6-2q & 0 & -6 \\ 6 & 0 & q & 8 & 6 \end{vmatrix} = 80$ **… [M1]**

$\dfrac{1}{2}\left[-4q + 8(6-2q) + 24 + 48\right] = 80$

	$\frac{1}{2}[-20q+120]=80$ $q=-2$ **… [A1]** When $q=-2$, $p=6-2(-2)=10$ Hence, $A=(10,-2)$ **… [A1]**