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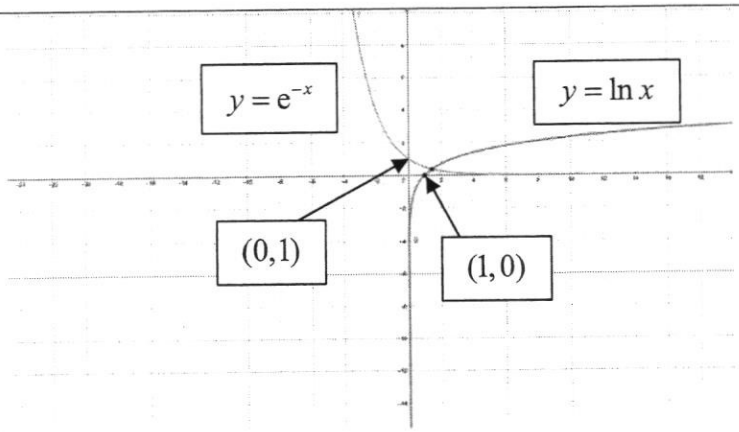
1	(i)				
	(ii)	1 solution			
2		Sub $y=2x+3$ into $y^2=x+3$ $4x^2+11x+6=0$ Alternatively Sub $x=\frac{y-3}{2}$ into $y^2=x+3$			
		$(x+2)(4x+3)=0$ alternatively $(2y-3)(y+1)=0$			
		$x=-2$ or $x=-\frac{3}{4}$ alternative $y=-1$ or $y=\frac{3}{2}$			
		$(-2,-1)$ and $\left(-\frac{3}{4},\frac{3}{2}\right)$			
3		$h=-2\left(t^2-\frac{3}{2}t\right)+c$ $=-2\left(t-\frac{3}{4}\right)^2+\frac{9}{8}+c$ $\therefore \frac{9}{8}+c>3$ $c>\frac{15}{8}$			
4		$\frac{1}{2}\times AC\times\left(4+8\sqrt{3}\right)\times\sin 60^\circ=24+15\sqrt{3}$			

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		$\frac{1}{2}\times AC\times\left(4+8\sqrt{3}\right)\times\left(\frac{\sqrt{3}}{2}\right)=24+15\sqrt{3}$ $AC=\frac{24+15\sqrt{3}}{6+\sqrt{3}}\times\frac{6-\sqrt{3}}{6-\sqrt{3}}$ $=\frac{144-24\sqrt{3}+90\sqrt{3}-45}{33}$ $=\left(3+2\sqrt{3}\right)$ cm			
5		$\frac{8x^2-3x-6}{(x-3)(2x^2+1)}=\frac{A}{x-3}+\frac{Bx+C}{2x^2+1}$ $A=3,\ B=2,\ C=3$ $\frac{8x^2-3x-6}{(x-3)(2x^2+1)}=\frac{3}{x-3}+\frac{2x+3}{2x^2+1}$			
6	(i)	Sub $t=0$ and $C=13$, $A=13$ Sub $t=18$ and $C=65\ 686$, $13e^{18k}=65686$			

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	(ii)	$18k = \ln\left(\frac{65686}{13}\right)$ $k = \frac{1}{18}\ln\left(\frac{65686}{13}\right)$ $= 0.4737352729$ $= 0.474$ (3sf) $13e^{kt} > 100000$ $t > \ln\left(\frac{100000}{13}\right) \div \frac{1}{18}\ln\left(\frac{65686}{13}\right)$ $t > 18.9$ (3sf) Minimum number of months = 19 months			
7	(i)	$y = -\frac{3}{4}x + 15$ $m_{AB} = \frac{4}{3}$			

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	(ii)	$y+1=\frac{4}{3}(x-8)$ $y=\frac{4}{3}x-\frac{35}{3}$ Sub $y=-\frac{3}{4}x+15$ into $y=\frac{4}{3}x-\frac{35}{3}$, $x=12.8\ \left(\text{or } \frac{64}{5}\right)$ Sub $x=12.8$ into $y=-\frac{3}{4}x+15$, $y=5.4\ \left(\text{or } \frac{27}{5}\right)$ $\therefore B(12.8,5.4)\ \left(\text{or } B\left(\frac{64}{5},\frac{27}{5}\right)\right)$ Area $=\frac{1}{2}\begin{vmatrix}0 & 8 & 12.8 & 0 & 0\\ 0 & -1 & 5.4 & 15 & 0\end{vmatrix}$ $=\frac{1}{2}\left[(43.2+192)-(-12.8)\right]$ or $=\frac{1}{2}\left[\left(\frac{216}{5}+192\right)-\left(-\frac{64}{5}\right)\right]$ $=124$ unit^2			
8	(a)	$\log_a a^5$ $=5$			

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	(b)	$\log_3 \frac{3}{p} + \frac{2\log_3 p}{\log_3 3^{-2}} = 3$ $\log_3 \frac{3}{p} - \log_3 p = 3$ $\log_3 \frac{3}{p^2} = 3$ $\frac{3}{p^2} = 3^3$ $p = \frac{1}{3}$ or $-\frac{1}{3}$ (rejected)			
9	(i)	$T_{r+1} = \binom{9}{r}\left(x^2\right)^{9-r}\left(-\frac{m}{x}\right)^r$ $= \binom{9}{r}(-m)^r x^{18-3r}$ $18 - 3r = 3$ $r = 5$			

		$\binom{9}{5}(-m)^5 = -\frac{14}{27}$ $m^5 = \frac{1}{243}$ $m = \sqrt[5]{\frac{1}{243}}$ $= \frac{1}{3}$ (Shown)			
	(ii)	$18-3r=0$ $r=6$ $T_7 = \binom{9}{6}\left(-\frac{1}{3}\right)^6$ $= \frac{28}{243}$ $\left(243+\frac{54}{x^3}\right)\left(x^2-\frac{m}{x}\right)^9 = \left(243+\frac{54}{x^3}\right)\left(\ldots-\frac{14}{27}x^3+\frac{28}{243}+\ldots\right)$ $= \ldots 243\left(\frac{28}{243}\right)+\left(\frac{54}{x^3}\right)\left(-\frac{14}{27}x^3\right)+\ldots$ $= \ldots+28-28+\ldots$ $= \ldots+0+\ldots$ Since the expansion gave a constant value of 0, there is no term independent in the expansion.			
10	(i)	$f(-2) = 2(-2)^3 - 5(-2)^2 - 9$			
		$= -45$			
	(ii)	$f(3) = 2(3)^3 - 5(3)^2 - 9$ $= 0$ By Factor Theorem, $(x-3)$ is a factor. (Shown)			

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		$\begin{array}{r} 2x^2+x+3 \\ x-3\overline{)\,2x^3-5x^2+0x-9} \\ \underline{-(2x^3-6x^2)+0x+0} \\ x^2-0x-9 \\ \underline{-(x^2-3x)+0} \\ 3x-9 \\ \underline{-(3x-9)} \\ 0 \end{array}$			
		$(x-3)(2x^2+x+3)=0$			
		$x=3$			
		Since discriminant of $2x^2+x+3$ is $-23<0$,			
		there are no real solutions for the quadratic equation, hence there is only 1 solution for the cubic equation.			
11	(i)	$m_{chord}=-2$ $m_{normal}=\frac{1}{2}$			
		Midpoint of $AB=(5,-1)$			
		Equation of perpendicular bisector: $y=\frac{1}{2}x-\frac{7}{2}$			

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		Solve $y=\frac{1}{2}x-\frac{7}{2}$ and $6y=x-15$ simultaneously, $C(3,-2)$			
	(ii)	radius $=5$			
		$(x-3)^2+(y+2)^2=25$			
	(iii)	$(x-5)^2+(y-6)^2=25$			
		Distance between the 2 centres $=\sqrt{68}<5+5$			
		Since the distance between the 2 centres is less than the sum of radii, the two circles will intersect.			
12	(a) (i)	$-\frac{12}{13}$			
	(ii)	$\left(-\frac{12}{13}\right)\left(-\frac{7}{25}\right)-\left(\frac{5}{13}\right)\left(\frac{24}{25}\right)$			
		$=-\frac{36}{325}$			
	(iii)	$2\cos^2\frac{A}{2}-1=-\frac{12}{13}$			

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		$\cos\frac{A}{2}=\sqrt{\frac{1}{26}}$ or $-\sqrt{\frac{1}{26}}$ (rejected)			
		$2\left(\sin\frac{A}{2}\right)\left(\sqrt{\frac{1}{26}}\right)=\frac{5}{13}$ $\sin\frac{A}{2}=\frac{5}{\sqrt{26}}$ $\left(\text{or } \frac{5\sqrt{26}}{26}\right)$			
	(iv)	$\dfrac{-\frac{5}{12}+\tan C}{1-\left(-\frac{5}{12}\right)\tan C}=-\frac{3}{41}$			
		$\tan C=\frac{1}{3}$			
	(b) (i)	3			
	(ii)	720° or 4π			

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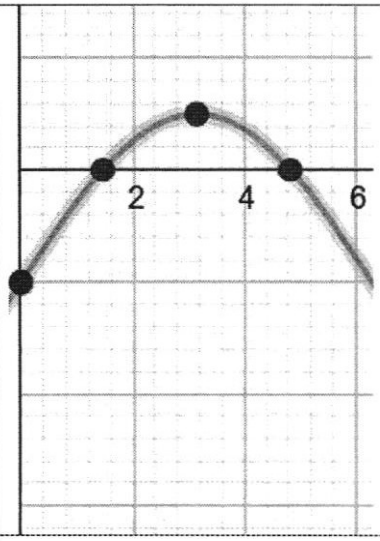
	(iii)				
13	(a) (i)	$LHS = \dfrac{1-2\sin^2\theta+1}{2(1-\sin\theta)}$			
		$= \dfrac{2(1-\sin^2\theta)}{2(1-\sin\theta)}$			
		$= \dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)}$ $= \sin\theta+1$ $= RHS$ (proven)			
	(ii)	$\dfrac{\cos 2\theta+1}{2-2\sin\theta} = \dfrac{2}{\sin\theta}$ $\sin\theta+1 = \dfrac{2}{\sin\theta}$ $\sin^2\theta+\sin\theta-2=0$			
		$(\sin\theta+2)(\sin\theta-1)=0$			
		$\sin\theta=1$ or $\sin\theta=-2$ (rejected)			
		$\therefore \theta = 90°$			

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	(b) (i)	$\frac{AB}{6}=\sin\theta$ and $\frac{BC}{4}=\cos\theta$ $AB=6\sin\theta$ and $BC=4\cos\theta$ $\therefore P=2(6\sin\theta+4\cos\theta)$ $=8\cos\theta+12\sin\theta$ (Shown)			
	(ii)	$R=\sqrt{8^2+12^2}$ $=\sqrt{208}=4\sqrt{13}$			
		$\alpha=\tan^{-1}\left(\frac{12}{8}\right)$ $=0.9827937232$ $=0.983$ (3sf)			
		$\therefore 8\cos\theta+12\sin\theta=4\sqrt{13}\cos(\theta-0.983)$			
		$4\sqrt{13}\cos(\theta-0.983)=10$ $\cos(\theta-0.983)=\frac{10}{4\sqrt{13}}$ $\alpha=\cos^{-1}\left(\frac{10}{4\sqrt{13}}\right)$ $=0.8046336771$			
		$\theta=0.1781600461$ or 1.7874274 (rejected) $=0.178$ (3sf)			